Algebraic and analytic reconstruction methods for dynamic tomography

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Dynamic tomography

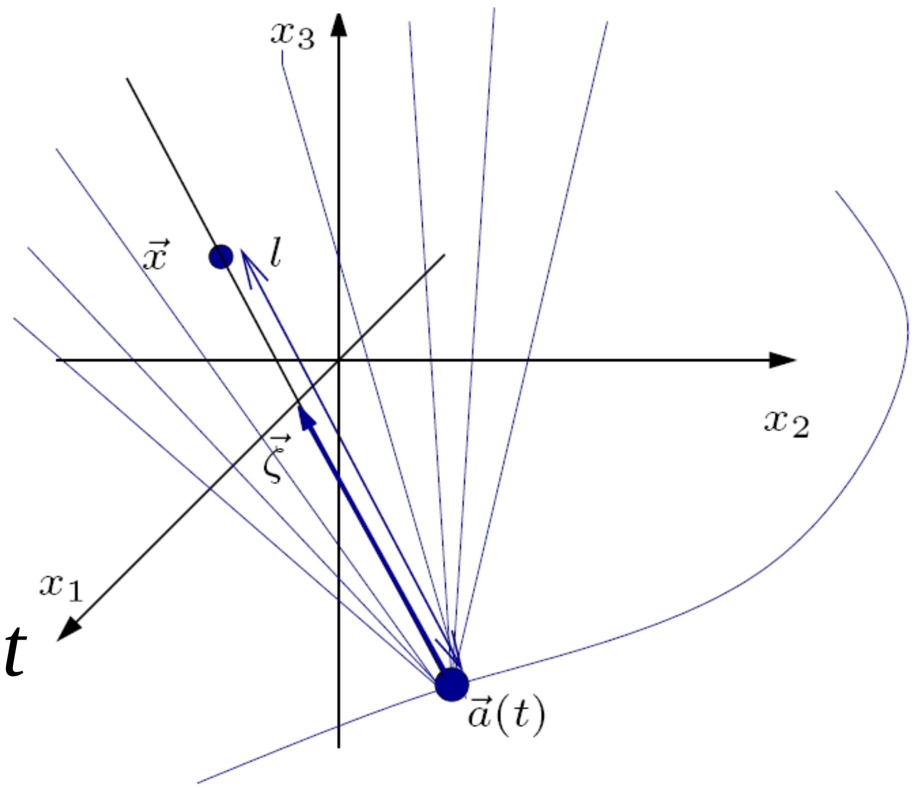
3D cone beam transform

$$g_{\mathcal{D}}(t,\vec{\zeta}) \stackrel{\text{def}}{=} \mathcal{D}f(t,\vec{\zeta}) \stackrel{\text{def}}{=} \mathcal{D}_t f(\vec{\zeta}) \stackrel{\text{def}}{=} \int_0^{+\infty} f\left(\vec{a}(t) + l\vec{\zeta}\right) dl$$

$\vec{\zeta} \in S^2$ is a unit vector in $\mathbb{R}^3$

time $t \in T \subset \mathbb{R}$

$\vec{a}(t) \in \mathbb{R}^3$ is the x-ray source position



a time dependent deformation model $\vec{\Gamma}_t$

In dynamic CT, the attenuation *f* is a function of *t*

=> We assume $f_{\vec{\Gamma}_t}(\vec{x}) = f\left(\vec{\Gamma}_t(\vec{x})\right)$

$\vec{\Gamma}_t$ are known bijective smooth functions on $\mathbb{R}^3$

Analytic dynamic reconstruction

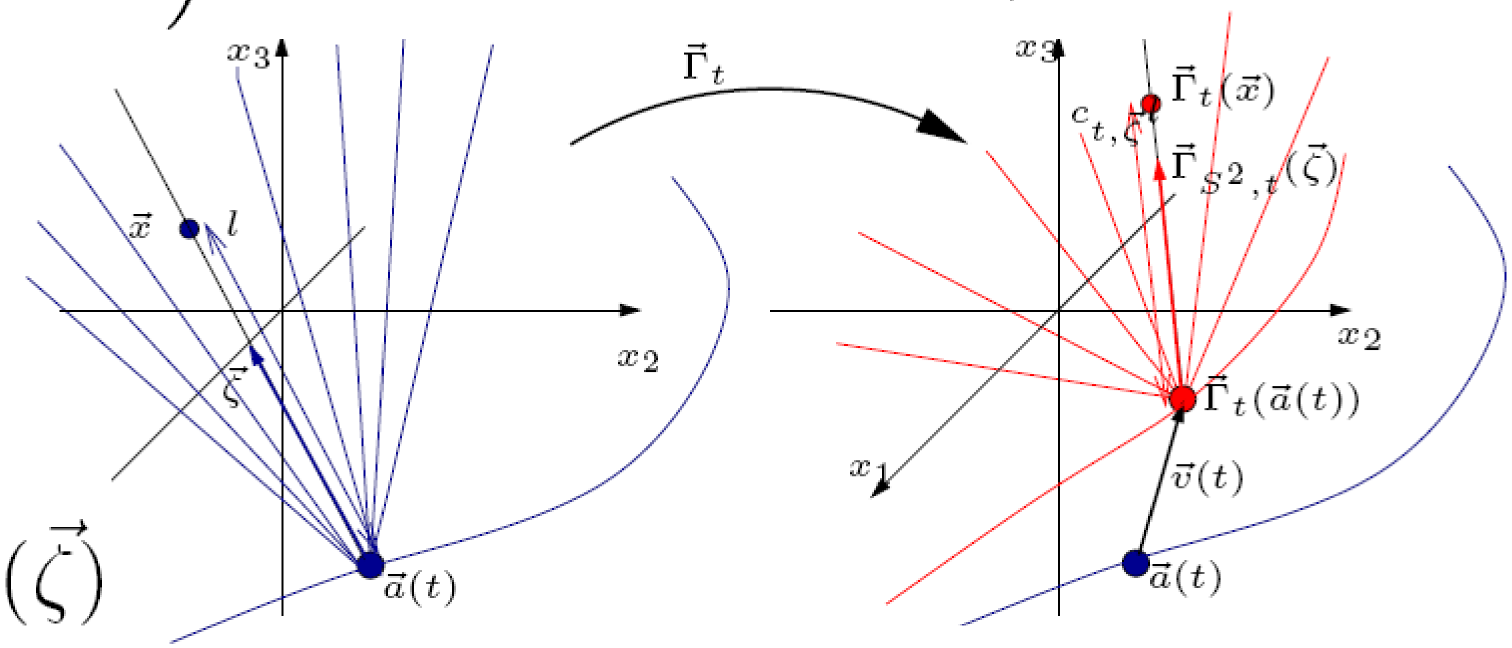
$\vec{\Gamma}_t$ leaves the CB geometry globally invariant if

$$\forall t \in T, \forall \vec{\zeta} \in S^2 \; \vec{\Gamma}_t\left(\vec{a}(t) + \mathbb{R}^+\vec{\zeta}\right) = \vec{\Gamma}_t\left(\vec{a}(t)\right) + \mathbb{R}^+\vec{\Gamma}_{S^2,t}(\vec{\zeta})$$

where $\vec{\Gamma}_{S^2,t} \quad S^2 \longrightarrow S^2$
is a diffeomorphism
on the unit sphere

$$\vec{\Gamma}_t\left(\vec{x}\right) = \vec{\Gamma}_t\left(\vec{a}(t) + l\vec{\zeta}\right)$$

$$= \vec{\Gamma}_t\left(\vec{a}(t)\right) + \Gamma_{t,\vec{\zeta}}(l)\vec{\Gamma}_{S^2,t}(\vec{\zeta})$$



let $\Gamma_{t,\vec{\zeta}}(l) = c_{t,\vec{\zeta}}\,l \quad c_{t,\vec{\zeta}} > 0$

let $\vec{v}(t) \stackrel{\text{def}}{=} \vec{\Gamma}_t\left(\vec{a}(t)\right) - \vec{a}(t)$

$$\vec{\Delta}_t\left(\vec{x}\right) = \vec{a}(t) + c_{t,\vec{\zeta}}\,l\vec{\Gamma}_{S^2,t}(\vec{\zeta})$$

$$\vec{T}_{\vec{v}(t)}(\vec{x}) = \vec{v}(t) + \vec{x}$$

$$\vec{\Gamma}_t = \vec{T}_{\vec{v}(t)} \circ \vec{\Delta}_t$$

$$\mathcal{D}_t f_{\vec{\Delta}_t}(\vec{\zeta}) = \int_0^{+\infty} f\left(\vec{a}(t) + c_{t,\vec{\zeta}}\,l\vec{\Gamma}_{S^2,t}(\vec{\zeta})\right) dl$$

$$= \frac{1}{c_{t,\vec{\zeta}}}\mathcal{D}_t f(\vec{\Gamma}_{S^2,t}(\vec{\zeta})).$$

Analytic compensation of Δ_t

$$\mathcal{D}_t f(\vec{\zeta}) = c_{t,\vec{\Gamma}^{-1}_{S^2,t}(\vec{\zeta})} \mathcal{D}_t f_{\vec{\Delta}_t}(\vec{\Gamma}^{-1}_{S^2,t}(\vec{\zeta}))$$

Thus $\mathcal{D}_t f_{\vec{T}_{\vec{v}(t)}}(\vec{\zeta}) = c_{t,\vec{\Gamma}^{-1}_{S^2,t}(\vec{\zeta})} \mathcal{D}_t f_{\vec{T}_{\vec{v}(t)} \circ \vec{\Delta}_t}(\vec{\Gamma}^{-1}_{S^2,t}(\vec{\zeta}))$

As
$$\begin{aligned} \mathcal{D}_t f_{\vec{T}_{\vec{v}(t)}}(\vec{\zeta}) &= \int_0^{+\infty} f\left(\vec{a}(t) + \vec{v}(t) + l\vec{\zeta}\right) dl \\ &= \int_0^{+\infty} f\left(\vec{\Gamma}_t\left(\vec{a}(t)\right) + l\vec{\zeta}\right) dl. \end{aligned}$$

The reconstruction of f from $\mathcal{D}_t f_{\vec{T}_{\vec{v}(t)}}(\vec{\zeta})$ is just the reconstruction on the virtual trajectory $\vec{\Gamma}_t(\vec{a}(t))$

Algebraic reconstruction

Let $(e_j)_{j \in J}$ be a set of basis functions $e_j : \mathbb{R}^3 \to \mathbb{R}$

$$f(\vec{x}) = \sum_{j \in J} f_j e_j(\vec{x}) \quad \text{where } f_j \in \mathbb{R}, j \in J$$

$$d_i = \int_\Omega h_i(\vec{x}) f(\vec{x}) d\vec{x}, i \in I$$

where $(d_i)_{i \in I}$ is the real vector of acquired data

$h_i(\vec{x})$ could be the dirac on a line $\delta(\vec{x} \cdot \vec{\theta}_i - s_i)$

could be the indicator of the conical region joining the source to the detector

$$\begin{aligned} d_i &= \int_\Omega h_i(\vec{x}) f(\vec{x}) d\vec{x} = \int_\Omega h_i(\vec{x}) \sum_{j \in J} f_j e_j(\vec{x}) d\vec{x} \\ &= \sum_{j \in J} \left(\int_\Omega h_i(\vec{x}) e_j(\vec{x}) d\vec{x} \right) f_j \end{aligned}$$

linear system $\mathbf{d} = \mathbf{A}\mathbf{f}$

$A_{i,j} = \int_\Omega h_i(\vec{x}) e_j(\vec{x}) d\vec{x}$

$\mathbf{d} = (d_i)_{i=1,\ldots,n_I}$ data

$\mathbf{f} = (f_j)_{j=1,\ldots,n_J}$ unknown coefficients

Algebraic dynamic reconstruction

Dynamic data
$$d_{t,i} = \int_\Omega h_i(\vec{x}) f\left(\vec{\Gamma}_t(\vec{x})\right) d\vec{x}$$
$$= \sum_{j\in J}\left(\int_\Omega h_i(\vec{x}) e_j\left(\vec{\Gamma}_t(\vec{x})\right) d\vec{x}\right) f_j$$

Assume their exists $(b_k)_{k\in K}$, K multi-index set, such that
$$e_j\left(\vec{\Gamma}_t(\vec{x})\right) = \sum_{k\in K} \Gamma_{t_{k,j}} b_k(\vec{x})$$

Then
$$d_{t,i} = \sum_{j\in J}\left(\int_\Omega h_i(\vec{x}) \sum_{k\in K} \Gamma_{t_{k,j}} b_k(\vec{x}) d\vec{x}\right) f_j$$
$$= \sum_{j\in J}\left(\sum_{k\in K} B_{i,k}\Gamma_{t_{k,j}}\right) f_j \qquad B_{i,k} = \int_\Omega h_i(\vec{x}) b_k(\vec{x}) d\vec{x}$$

$d_{t,i}$ is $d_{t(i_1),i_2}$, where $i = (i_1, i_2)$ $\mathbf{d_{i_1}} = \mathbf{B_{i_1}\Gamma_{t(i_1)}f}, i_1 = 1,\ldots,n_{I_1}$

Numerical experiments

The phantom was obtained from a 4D CT image acquired on a scanner synchronized with a respiratory signal [24]. Motion vector fields were computed by deformable registration between each 3D CT image and the reference (one among the ten 3D CT used). A set of cone-beam projections were computed using a projector taking into account this motion model and the geometry of an existing cone-beam CT scanner (640 projections, 512x512 pixels of 0.52x0.52 cm² at isocenter, source/center distance 1000 mm ; source/detector 1536mm, circular trajectory).

Reconstruction methods

Discussion

- Both algebraic and analytic methods were presented for dynamic tomography
- Algebraic approaches are « slow » but better suited for general deformations
- Next work : application and comparisons on real data

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