(Circular) Tomosynthesis
and
(Random Conical Tilt) Transmission Electron Microscopy

Formulas given during the "ToRIID" seminar of March 3, 2009 at St.Jean des Vignes

Forward model:

$$p_\phi(u, v) = \int f(t\underline{n} + u\underline{\alpha} + v\underline{\beta})dt$$

$$\underline{n} = (\sin\theta_0\cos\phi, \sin\theta_0\sin\phi, \cos\theta_0)$$

$$\underline{\alpha} = (-\sin\phi, \cos\phi, 0)$$

$$\underline{\beta} = (-\cos\phi, -\sin\phi, 0)$$

Backprojection (= "tomosynthesis"):

$$b(\underline{x}) = \mathcal{B}p(\underline{x}) = \int_0^{2\pi} p_\phi(u, v)\Big|_{\substack{u = \underline{x}\cdot\underline{\alpha} \\ v = \underline{x}\cdot\underline{\beta}^\perp/(\cos\theta_0)}} d\phi$$

$$\underline{\beta}^\perp = (-\cos\theta_0\cos\phi, -\cos\theta_0\sin\phi, \sin\theta_0)$$

(It may be helpful to note that $v\cos\theta_0 = v^\perp$.)

Fourier-slice theorem:

$$P_\phi(\nu_u, \nu_v) = \left(\frac{1}{\cos\theta_0}\right)F(\nu_u\underline{\alpha} + \nu_v\underline{\beta}^\perp/(\cos\theta_0))$$

Filtered backprojection (FBP):

$$f_R(\underline{x}) = \mathcal{B}p^F(\underline{x}) = \int_0^{2\pi} p_\phi^F(u, v)\Big|_{\substack{u = \underline{x}\cdot\underline{\alpha} \\ v = \underline{x}\cdot\underline{\beta}^\perp}} d\phi$$

$$P_\phi^F(\nu_u, \nu_v) = (\cos\theta_0)P_\phi(\nu_u, \nu_v\cos\theta_0)\ |\nu_u|$$

(Note: p^F is not in tomosynthesis format but in the tilted (perpendicular) plane)

$$F_R(\underline{\nu}) = \begin{cases} F(\underline{\nu}) & \text{if } \frac{|\nu_3|}{\|\underline{\nu}\|} < \sin\theta_0 \\ 0 & \text{otherwise (in missing cone)} \end{cases}$$

whereas

$$B(\underline{\nu}) = \begin{cases} \frac{F(\underline{\nu})D(\theta_0)}{\|\underline{\nu}\|} & \text{if } \frac{|\nu_3|}{\|\underline{\nu}\|} < \sin\theta_0 \\ 0 & \text{otherwise (in missing cone)} \end{cases}$$

for some angle-dependent function $D(\theta_0)$.

Now, I need some diagrams, and the proofs of all this. Rolf. March 5/09