Analytic reconstruction methods for the Compton camera

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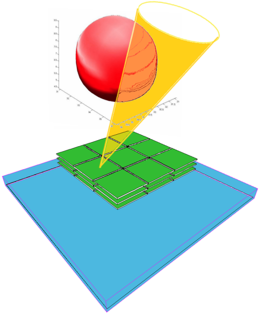
Grenoble, September 10-11, 2013

Summary

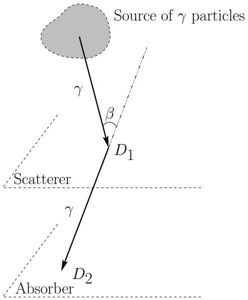
Summary

Compton camera for SPECT imaging



- Source of γ particles : emission point V_0 and initial energy E
- Scatterer : first interaction (Compton scattering) at V_1 and energy transmitted to an electron E_1
- Absorber : second interaction at V_2 (photoelectric absorption) and energy E_2
- Projection pattern : integral on the surface of a cone

The data



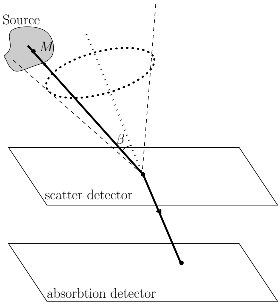
The diffusion angle, also called Compton angle, is then given by

$$\cos\beta = 1 - \frac{m_e c^2 E_1}{(E - E_1)E}$$

A γ particle emitted at V_0 with initial energy E

- is Compton scattered at V_1 where the energy E_1 is transferred to an electron of the scatterer
- then is absorbed by photoelectric effect at the point V_2 from the absorber, where the remaining energy $E_2 = E - E_1$ is deposed.

The data



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Conversely, the source point V_0 lies on the surface of a cone, the Compton cone, with

- apex V_1
- axis direction $\overrightarrow{\Omega_2} = \frac{\overrightarrow{V_2 V_1}}{V_2 V_1}$
- half-opening angle β

Applications

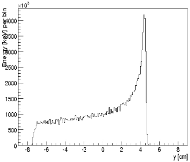
- Imaging of polyenergetic sources
- Imaging of sources with energies ~ 1 MeV

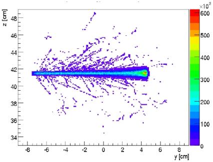
Advantages of the Compton camera :

- devoided of mechanical collimator, its sensitivity is superior to the one of the Anger camera by 1-2 orders of magnitude
- 3D imaging with a single camera

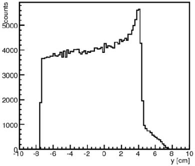
Example of application : hadron therapy

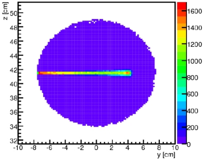
Sphere in PMMA irradiated by a proton beam (140 MeV).





Deposited energy



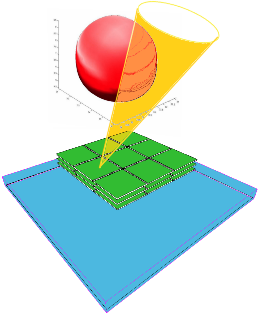


Last interaction of γ particles that escape the phantom.

("A tracking Compton-scattering imaging system for hadron therapy monitoring", M. Frandes, A. Zoglauer, V. Maxim, R. Prost, IEEE TNS, 2010)

Conclusions

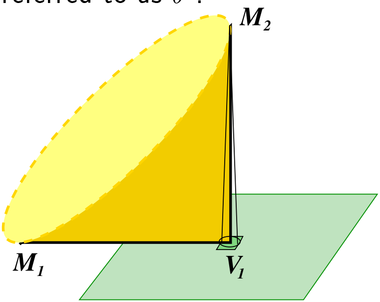
- Sources of γ particles are imaged,
- with a detector capable to detect particles from arbitrary directions,
- the measured projections being integrals, of the intensity of the source, on conical surfaces.



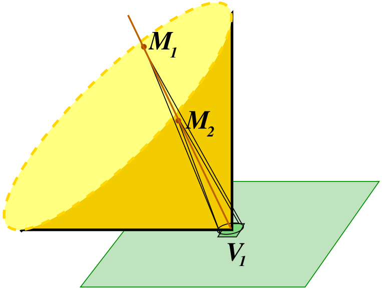
Summary

Important parameters

Influence of the angle of incidence, referred to as θ :



Influence of the distance to the detector :



Let us denote

- $\vec{v}$ the vector of coordinates of V_1 in the orthogonal frame $Oxyz$,
- $\vec{u}$ the vector of coordinates of an arbitrary point M
- θ the angle of incidence on the scatterer, $\cos\theta = \frac{(\vec{u}-\vec{v}).\vec{n}}{\|\vec{u}-\vec{v}\|}$.

The intensity of the source should be weighted by a function

$$h : (\theta, \|\vec{u}-\vec{v}\|) \mapsto h(\theta, \|\vec{u}-\vec{v}\|) = \frac{\cos\theta}{V_1M^2}.$$

General model

Let f be the intensity of the source. The number of γ particles scattered at V_1, with an angle β and absorbed at V_2 is proportional to :

$$\mathscr{C}(\vec{u}, \overrightarrow{\Omega_2}, \beta) = \int_{\mathbb{R}^3} f(\vec{u}) h(\theta, \|\vec{v} - \vec{u}\|) k(\vec{u}; \vec{v}, \overrightarrow{\Omega_2}, \beta) d\vec{u},$$

where $k(\vec{u}; \vec{v}, \overrightarrow{\Omega_2}, \beta)$ models the uncertainties on the value of β and may include the Klein-Nishina differential cross-section $K(\cos\beta)$.

Example : no uncertainties and $h(\theta, \|\vec{v} - \vec{u}\|) = 1$

- May be found in [Cree and Bones, 1994], [Basko et al 1998], [Smith 2005].
- The errors on the measures are not accounted for.
- Each point from the scatterer is seen as isolated.

Let us consider spherical coordinates in a local frame with the vertical axis directed by $\overrightarrow{\Omega_2}$. For $\phi \in [0, 2\pi)$, let us denote $\overrightarrow{\Omega_1} = \overrightarrow{\Omega_1}(\beta, \varphi)$ the generatrices of a Compton cone $C(V_1, V_2, \beta)$.

The Compton projections are then :

$$\mathscr{C}(\vec{u}, \overrightarrow{\Omega_2}, \beta) = K(\cos\beta)\sin\beta \int_0^{2\pi} \int_0^{\infty} f(\vec{u} + \rho\overrightarrow{\Omega_1})\rho d\rho d\varphi.$$

Relation to the 3D Radon transform (and redundancy)

Note that with this model,

$$\mathscr{C}(\vec{u}, \overrightarrow{\Omega_2}, \pi/2) = K(0)\mathscr{R}_3(\overrightarrow{\Omega_2}, \vec{v}.\overrightarrow{\Omega_2}),$$

where $\mathscr{R}_3$ denotes the three-dimensional Radon transform.

Example : no uncertainties and $h(\theta, \|\vec{v}-\vec{u}\|) = 1/\|\vec{v}-\vec{u}\|$

- May be found in [Parra, 2000], [Tomitani and Hirasawa 2002], [Smith 2005].
- The errors on the measures are not accounted for.
- Each point from the scatterer is seen as isolated.

The Compton projections are then :

$$\mathscr{C}(\vec{u}, \overrightarrow{\Omega_2}, \beta) = K(\cos\beta)\sin\beta \int_0^{2\pi}\int_0^{\infty} f(\vec{v} + \rho\overrightarrow{\Omega_1})d\rho d\varphi,$$

Relation to cone-beam integrals

$$p(\vec{v}, \overrightarrow{\Omega_1}) = \int_0^{\infty} f(\vec{v} + \rho\overrightarrow{\Omega_1})d\rho$$

are cone-beam integrals of the object. The Compton projections are in this case sum of cone-beam integrals.

Example : no uncertainties and $h(\theta, \|\vec{v} - \vec{u}\|) = \cos\theta$

- May be found in [Maxim et al, 2009].
- The errors on the measures are not accounted for.
- ~~Each point from the scatterer is seen as isolated.~~

The Compton projections are then :

$$\mathscr{C}(\vec{v}, \overrightarrow{\Omega_2}, \beta) = K(\cos\beta) \int_{(\vec{u}-\vec{v}).\overrightarrow{\Omega_2} = \|\vec{u}-\vec{v}\| \cos\beta} f(\vec{u}) \cos\theta d\vec{u}.$$

Example : The uncertainties are modelled and $h(\theta, \|\vec{v}-\vec{u}\|) = 1/\|\vec{v}-\vec{u}\|^2$

- May be found in [Hirasawa and Tomitani, 2003].
- Gaussian model for the uncertainties, $\beta \in [\beta_1, \beta_2]$,

$$k(\vec{u}; \vec{v}, \overrightarrow{\Omega_2}, \beta) = K(\cos\beta)\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(\cos(\widehat{\vec{u}-\vec{v}, \overrightarrow{\Omega_2}}) - \cos\beta)^2}{2(\sigma\sin\beta)^2}\right).$$

The Compton projections are then :

$$\mathscr{C}(\vec{v}, \overrightarrow{\Omega_2}, \beta) = \int_{\beta_1}^{\beta_2}\left(\int_0^{2\pi} p(\vec{v}, \overrightarrow{\Omega_1})d\varphi\right) k(v + \overrightarrow{\Omega_2}(\widetilde{\beta}, 0); \vec{v}, \overrightarrow{\Omega_2}, \beta)\sin\widetilde{\beta}d\widetilde{\beta},$$

Relation to cone-beam integrals

The Compton projections are again sum of cone-beam integrals.

Summary

State of the art

- Methods where the 3D Radon projections are calculated : [Basko et al, 1998], [Smith, 2005]
- Methods where the cone-beam projections are calculated : [Parra, 2000], [Tomitani and Hirasawa, 2002], [Hirasawa and Tomitani, 2003]
- Direct inversion : [Cree and Bones, 1994], [Maxim et al, 2009], [Lojacono et al, 2011], [Maxim, 201 ?]
- Methods using series expansions in spherical harmonics and Lagrange polynomials : [Basko et al, 1998], [Parra, 2000], [Tomitani and Hirasawa, 2002], [Hirasawa and Tomitani, 2003]
- Methods using the Hilbert transform : [Smith, 2005]
- Methods that may be compared to the Fourier-slice theorem and FBP : [Cree and Bones, 1994], [Maxim et al, 2009], [Lojacono et al, 2011], [Maxim, 201 ?].

[Tomitani and Hirasawa, 2002]

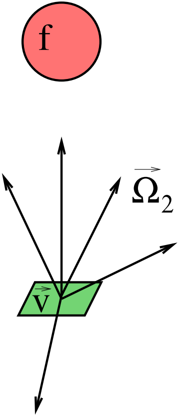
- The Compton projections

$$\mathscr{C}(\vec{v}, \overrightarrow{\Omega_2}, \beta) = K(\cos\beta)\sin\beta \int_0^{2\pi}\int_0^{\infty} f(\vec{v} + \rho\overrightarrow{\Omega_1})d\rho d\varphi,$$

are sums of cone-beam integrals

$$p(\vec{v}, \overrightarrow{\Omega_1}) = \int_0^{\infty} f(\vec{v} + \rho\overrightarrow{\Omega_1})d\rho.$$

- The aim of the method is to calculate, independently for each given $\vec{v}$, the values $p(\vec{v}, \overrightarrow{\Omega_1})$ for $\overrightarrow{\Omega_1} \in S$.
- Equivalent to filtering after back-projection on conical surfaces.



[Tomitani and Hirasawa, 2002] : reconstruction of cone-beam projections

The reconstruction formula is :

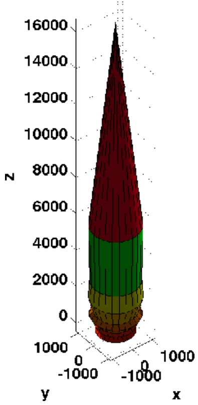
$$p(\vec{v},\overrightarrow{\Omega_1}) = \int_{\beta_1}^{\beta_2}\int_S k^{-1}(\overrightarrow{\Omega_2}.\overrightarrow{\Omega_1}), \cos\tilde{\beta})\mathscr{C}(\vec{v},\overrightarrow{\Omega_2},\tilde{\beta})\, d\overrightarrow{\Omega_2}\, d(\cos\tilde{\beta}),$$

with

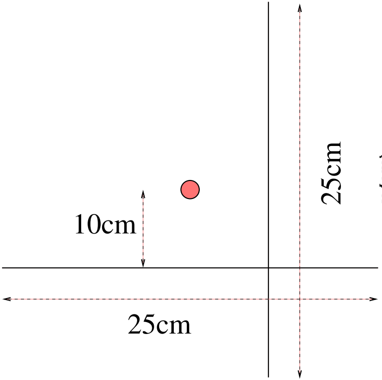
$$k^{-1}(s,t) = \frac{1}{4\pi}\sum_{n=0}^{\infty}\frac{2n+1}{H_n}P_n(s)P_n(t),\ s,t\in[-1,1]$$

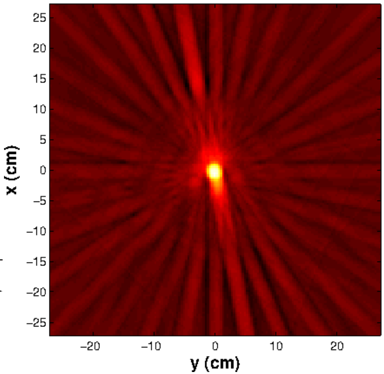
where P_n are Lagrange polynomials and

$$H_n = \int_{\beta_1}^{\beta_2} K(\cos\beta)P_n^2(\cos\beta)d(\cos\beta).$$



[Tomitani and Hirasawa, 2002] :
Reconstruction of one slice from a spherical source :





[Smith, 2005]

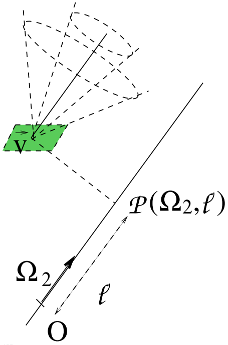
With

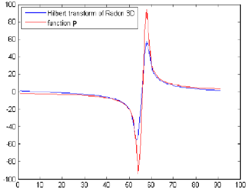
$$\mathscr{P}(\overrightarrow{\Omega_2}, \vec{v}.\overrightarrow{\Omega_2}) = -p.v. \int_0^{\pi} \mathscr{C}(\vec{v}, \overrightarrow{\Omega_2}, \beta) \frac{1}{\cos\beta} d\beta,$$

one has :

$$\mathscr{P}(\overrightarrow{\Omega_2}, \ell) = p.v. \int_{-\infty}^{\infty} \mathscr{R}_3(\overrightarrow{\Omega_2}, t) \frac{1}{\ell - t} dt,$$

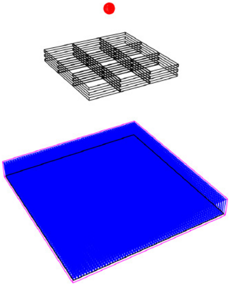
which is the Hilbert transform of the 3D Radon transform $\mathscr{R}_3(\overrightarrow{\Omega_2}, \cdot)$.

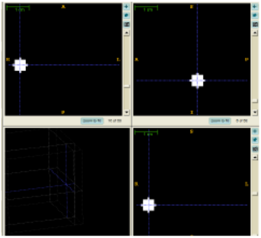




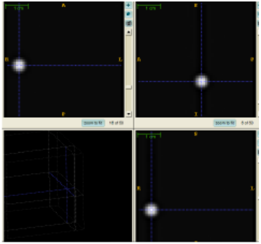
[Smith, 2005] : some results

Detector in 4π, deterministic projections :

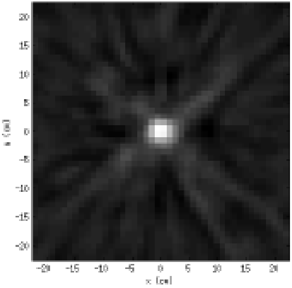




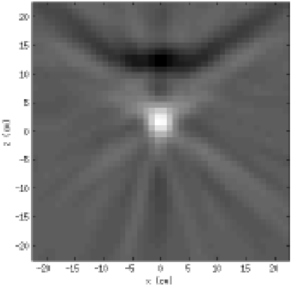
(a) original image



(b) reconstructed image



slice at z=0



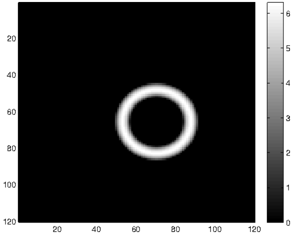
slice at y=0

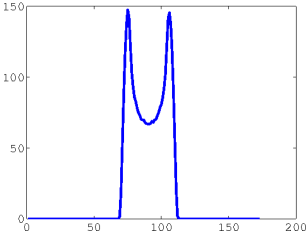
(MSc thesis of Hussein Banjak, co-supervised by Rolf Clackdoyle)

[Maxim 201 ?]

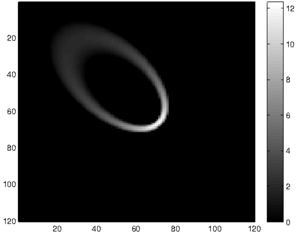
$$\mathscr{C}(\vec{v}, \overrightarrow{\Omega_2}, \beta) = K(\cos\beta) \int_{(\vec{u}-\vec{v}).\overrightarrow{\Omega_2} = \|\vec{u}-\vec{v}\| \cos\beta} f(\vec{u}) \cos\theta \, d\vec{u},$$

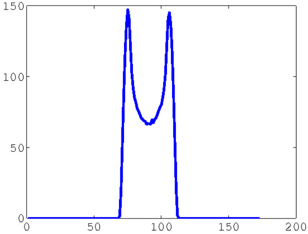
$\overrightarrow{\Omega_2}(0, \pi/4)$,
$\beta = \pi/3$
V_1 in the plane $z = 0$





$\overrightarrow{\Omega_2}(\pi/5, \pi/4)$,
$\beta = \arcsin(\frac{\sqrt{3}}{2} \cos\frac{\pi}{5})$
V_1 in the plane $z = 0$





[Maxim 201 ?] : Inversion

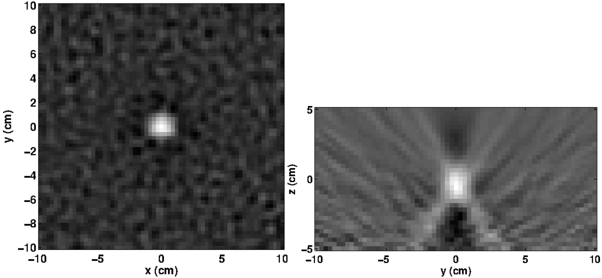
We show that

$$f(x,y,z) = 2\pi \int_0^{\pi} \int_{-\infty}^{\infty} \left(\int_0^{\infty} \widehat{\mathscr{P}_{\tau,\delta} f}(\rho) J_0(2\pi z \tau \rho) d\tau \right) e^{2i\pi\rho(-x\sin\delta + y\cos\delta)} |\rho|^3 d\rho d\delta,$$

with

$$\mathscr{P}_{\tau,\delta} = \frac{1}{K(\cos\beta)} \mathscr{R}_{\delta+\frac{\pi}{2}} \mathscr{C}(\cdot, \Omega(\alpha,\delta), \beta) \quad \text{for } \tau = \frac{\sin\beta}{\sqrt{\cos^2\alpha - \sin^2\beta}},$$

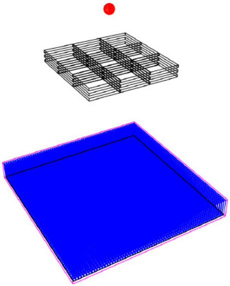
and J_0 the Bessel function.

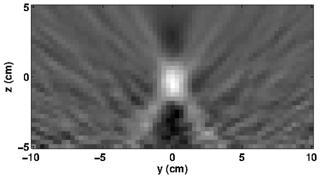


Summary

Redundancy in the data set

- The image space is three-dimensional.
- The data space is six-dimensional : 3 dof for the apex, 2 dof for the axis, 1 dof for the Compton angle.
- However, for a common acquisition geometry, not all the data are acquired, leading to artifacts in the reconstructed images.





Conclusions

- Several classes of methods were proposed in the literature.
- The Compton data are redundant but ...
- Except for maybe unpractical acquisition geometries, all the methods suffer from the absence of a part of the data sets.

Open questions :

- Is there any possibility for a local method, knowing that Compton projections are surface projections like the 3D Radon transform ?
- There are several models for the direct problem. Which one would be the best ?

Summary

This work was done with the contribution of :

- Xavier Lojacono (PhD student 2010-2013, CREATIS)
- Hussein Banjak (MSc student 2013, CREATIS)
- Estelle Hilaire (PhD student 2012-2015, CREATIS)
- Remy Prost (CREATIS)
- Rolf Clackdoyle (LHC)
- Françoise Peyrin (CREATIS)