2D fan-beam CT with independent source and detector rotation

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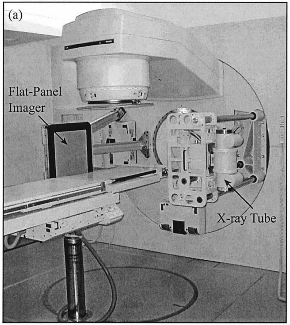
Image Guided Radiotherapy (IGRT)

- Imaging in the radiotherapy room
  - Fluoroscopy
  - Portal imaging
  - CT on rail
  - Ultrasound probe
  - ...
- Cone-beam CT since 10 years
  - Treatment guidance
  - Retrospective studies
  - Adaptive Radiotherapy

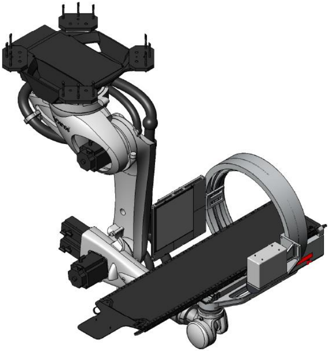


[Jaffray *et al.*, IJROBP, 2002]

New PAIR device (Salzburg)

Patient **A**lignment system with an integrated x-ray **I**maging **R**ing

- Ceiling mounted robotic arm
- Independent rotation of the source and the flat panel
- Couch translation
- Source collimation with 4 motorized jaws
- Fast switching between energies
- 41×41 cm^2 flat panel



medPhoton G.m.b.H
(courtesy of P. Steininger)

Installation: 1 prototype in Salzburg, 4 planned at MedAustron

Geometry in this presentation

[Gullberg *et al.*, IEEE TMI, 1986]

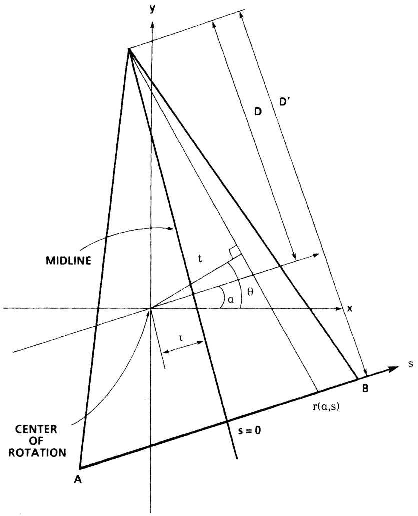
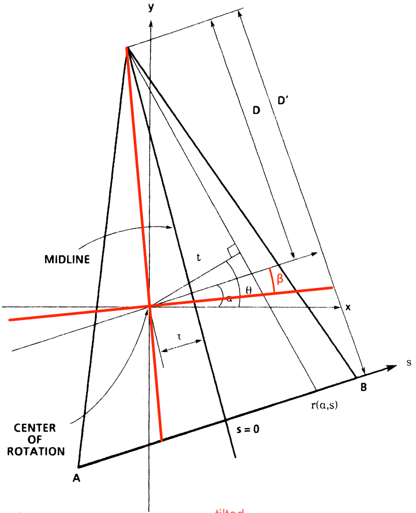


Fig. 2. Fan beam geometry with a displaced center-of-rotation for a flat detector.

Geometry in this presentation

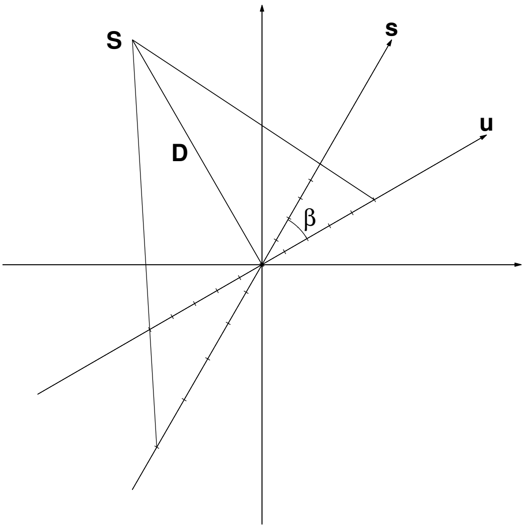
[Gullberg *et al.*, IEEE TMI, 1986]



tilted

Fig. 2. Fan beam geometry with a ~~displaced center of rotation for a~~ flat detector.

Effect of tilt on sampling



Effect of tilt on sampling

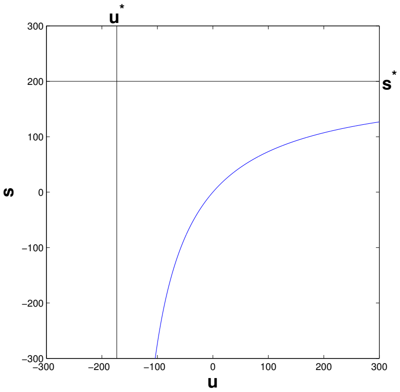
Relationship

$$s = \frac{uD}{D\cos\beta + u\sin\beta} \qquad (1)$$

Limits

$$\begin{cases} u^* = \dfrac{-D}{\tan\beta} \\[2ex] s^* = \dfrac{D}{\sin\beta} \end{cases} \qquad (2)$$

$D = 100,\ \beta = 30$



Effect of tilt on sampling

Derivative

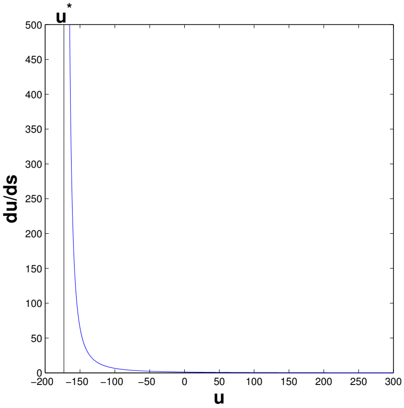
$$\frac{ds}{du} = \frac{D^2 \cos\beta}{(D\cos\beta + u\sin\beta)^2} \quad (3)$$

At origin,

$$\frac{ds}{du}(0) = \frac{1}{\cos\beta} \quad (4)$$

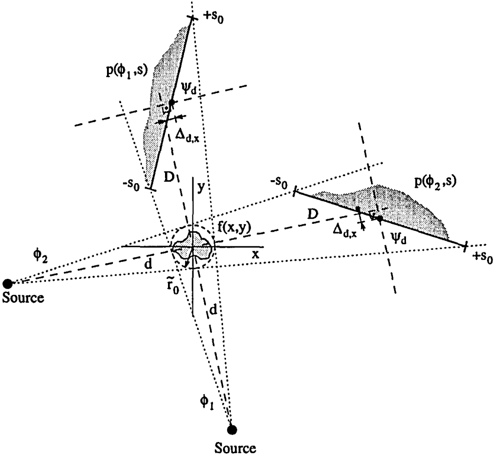
which would also be the constant sampling ratio in the parallel situation.

$D = 100,\ \beta = 30$



Potential use to increase spatial resolution

[Müller and Arce,
J Opt Soc Am A, 1994]



Inversion of the Radon transform

From parallel projections $p_p(\theta, t)$, the Fourier slice theorem leads to the inversion formula

$$f(r, \phi) = \int_0^{2\pi} \int_{-R}^{R} p_p(\theta, t) h[r\cos(\theta - \phi) - t]\, \mathrm{d}t\, \mathrm{d}\theta \qquad (5)$$

with

$$h(t) = \int_{\mathbb{R}} \frac{|\mu|}{2} \exp^{2i\pi\mu t} \mathrm{d}\mu. \qquad (6)$$

Implementation: filtered backprojection algorithm.

Change of variable [Gullberg *et al.*, TMI, 1986]

Assuming a flat detector at the origin (D=D'), we have

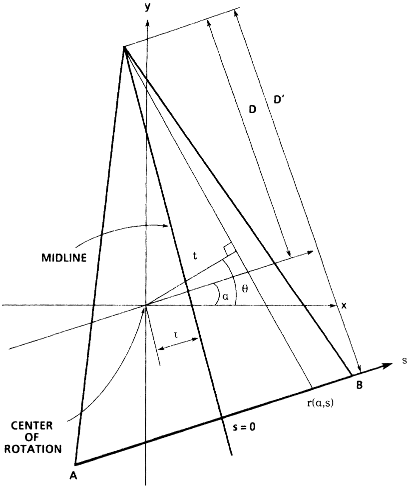
$$p_p(\theta, t) = p_f(\alpha, s) \qquad (7)$$

for

$$\begin{cases} t = (s+\tau)Z \\ \theta = \alpha + \tan^{-1}\left(\frac{s}{D}\right) \end{cases} \qquad (8)$$

with

$$Z = \frac{D}{\sqrt{s^2 + D^2}} \qquad (9)$$



Change of variable [Gullberg *et al.*, TMI, 1986]

Assuming that D and τ are constant, the Jacobian matrix is

$$\frac{\mathrm{d}t}{\mathrm{d}s} = (D^2 - \tau s)\frac{Z^3}{D^2} \tag{10}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}s} = \frac{Z^2}{D} \tag{11}$$

$$\frac{\mathrm{d}t}{\mathrm{d}\alpha} = 0 \tag{12}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}\alpha} = 1 \tag{13}$$

so its determinant is

$$J = \left|\left(D^2 - \tau s\right)\frac{Z^3}{D^2}\right| \tag{14}$$

Change of variable [Gullberg *et al.*, TMI, 1986]

$$f(r,\phi) = \int_0^{2\pi}\int_{-W}^{W} p_f(\alpha,s) h\left[r\cos(\alpha+\tan^{-1}(\frac{s}{D})-\phi)-(s+\tau)Z\right](D^2-\tau s)\frac{Z^3}{D^2}\,\mathrm{d}s\,\mathrm{d}\alpha$$

$$= \int_0^{2\pi}\int_{-W}^{W} p_f(\alpha,s) h\left[UZ(s'-s)\right](D^2-\tau s)\frac{Z^3}{D^2}\,\mathrm{d}s\,\mathrm{d}\alpha \tag{15}$$

with

$$U = \frac{r\sin(\alpha-\phi)+D}{D} \tag{16}$$

$$s' = \frac{rD\cos(\alpha-\phi)-\tau D}{r\sin(\alpha-\phi)+D} \tag{17}$$

Change of variable [Gullberg *et al.*, TMI, 1986]

We can then use an essential property of the filter:

$$h(at) = \frac{1}{a^2} h(t) \tag{18}$$

to obtain

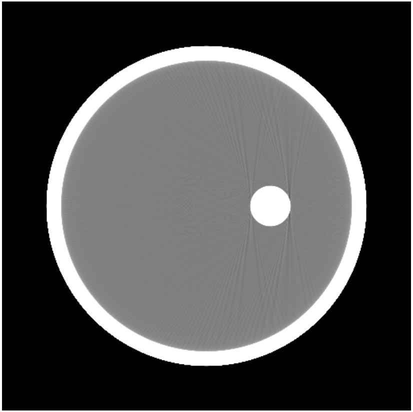
$$f(r,\phi) = \int_0^{2\pi} \frac{1}{U^2} \int_{-W}^{W} p_f(\alpha, s) \frac{D - \frac{\tau s}{D}}{\sqrt{s^2 + D^2}} h(s' - s)\, \mathrm{d}s\, \mathrm{d}\alpha \tag{19}$$

Experiments [Gullberg *et al.*, TMI, 1986]

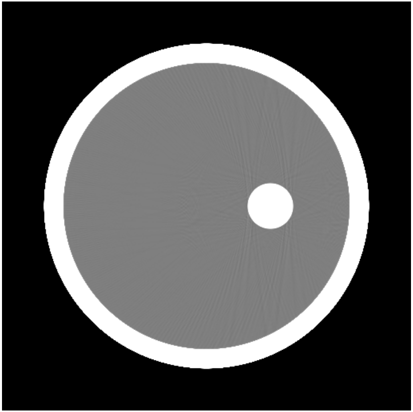
- $D = 630\,\mathrm{mm}$, $D' = 1100\,\mathrm{mm}$, $\tau = 1\,\mathrm{mm}$
- Projections
  - 1000
  - 768 samples
  - $0.2\,\mathrm{mm}$ spacing
- Reconstruction
  - 512×512 pixels
  - $0.125 \times 0.125\,\mathrm{mm}^2$ spacing

Experiments [Gullberg *et al.*, TMI, 1986]

Gray level window: [-582, -482] HU.



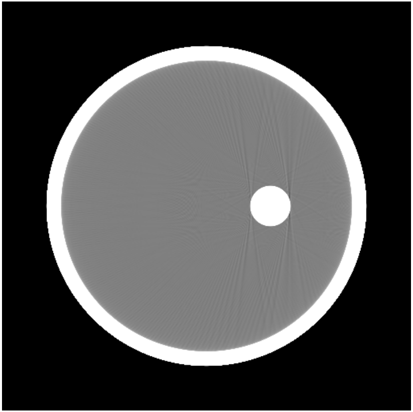
$\tau = 0\,\mathrm{mm}$



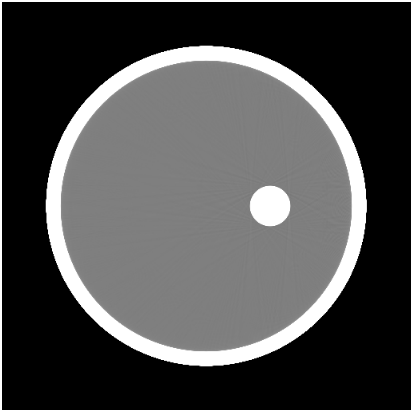
$\tau = 1\,\mathrm{mm}$, no correction

Experiments [Gullberg *et al.*, TMI, 1986]

Gray level window: [-582, -482] HU.



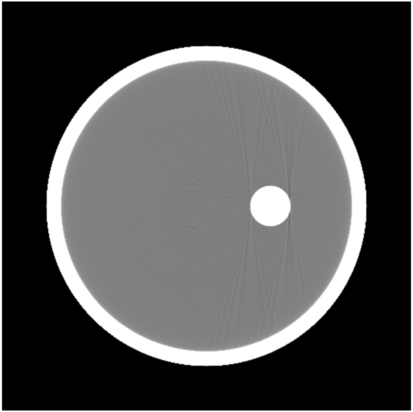
$\tau = 0\,\mathrm{mm}$



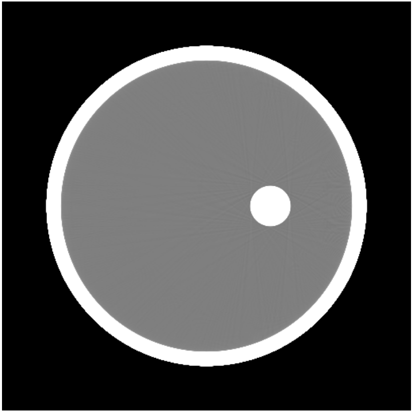
$\tau = 1\,\mathrm{mm}$, new algorithm

Experiments [Gullberg *et al.*, TMI, 1986]

Gray level window: [-582, -482] HU.

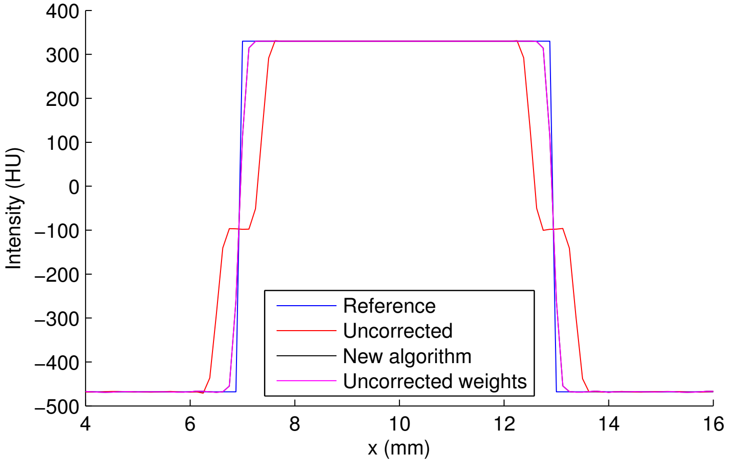


$\tau = 0\,\mathrm{mm}$



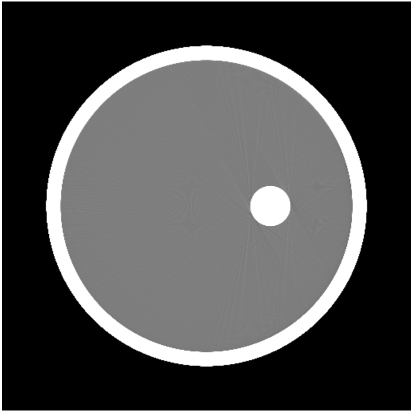
$\tau = 1\,\mathrm{mm}$, uncorrected weights

Experiments

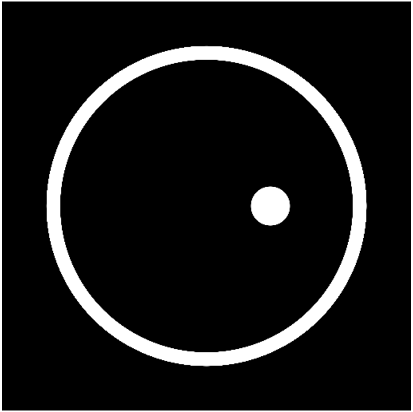


Experiments

Gray level window: [-582, -482] HU.

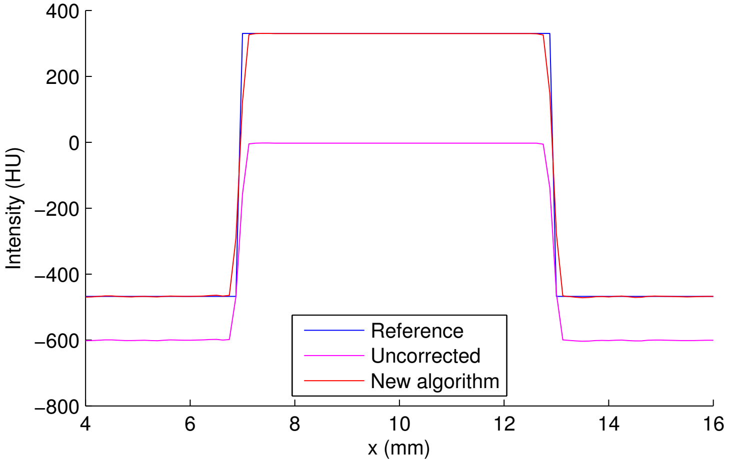


$\beta = 30^\circ$, new algorithm



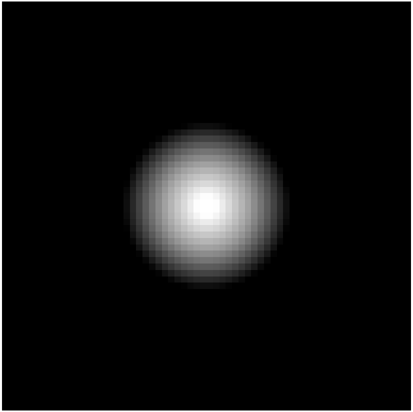
$\beta = 30^\circ$, uncorrected weights

Experiments



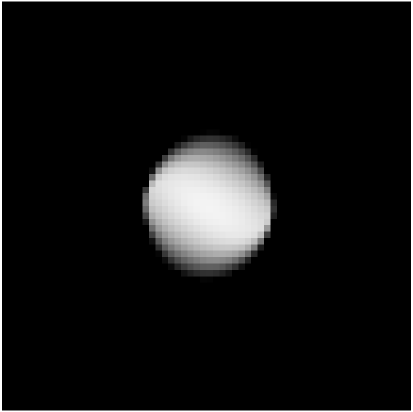
Point Spread Function

- $D = \tau = 1000\,\mathrm{mm}$, $\beta = 45^\circ$
- Ball at (0, 0), radius 0.01 mm, density 1000 HU
- Projections
  - 1000 projections
  - 10 mm spacing
  - 1000 rays per pixel
- Reconstruction
  - Centered on (0, 0)
  - 64×64 pixels
  - $1 \times 1\,\mu\mathrm{m}^2$ spacing



Point Spread Function

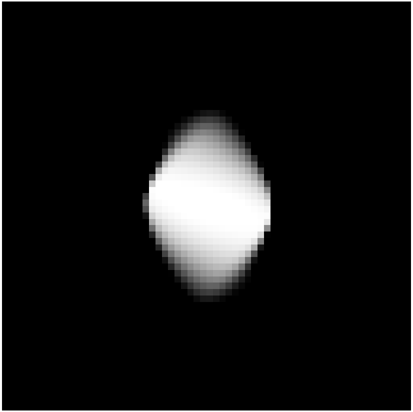
- $D = \tau = 1000\,\mathrm{mm}$, $\beta = 45°$
- Ball at (800, 0), radius $0.01\,\mathrm{mm}$, density 1000 HU
- Projections
  - 1000 projections
  - $10\,\mathrm{mm}$ spacing
  - 1000 rays per pixel
- Reconstruction
  - Centered on (800, 0)
  - 64×64 pixels
  - $1 \times 1\,\mu\mathrm{m}^2$ spacing



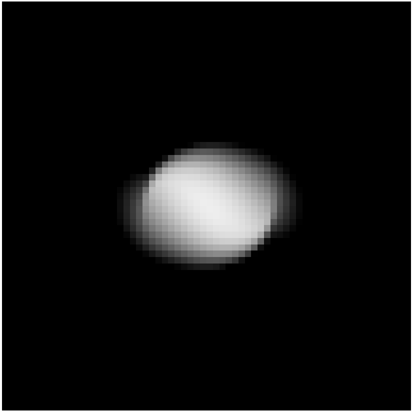
Point Spread Function

Short scan [Parker, Med Phys, 1982]

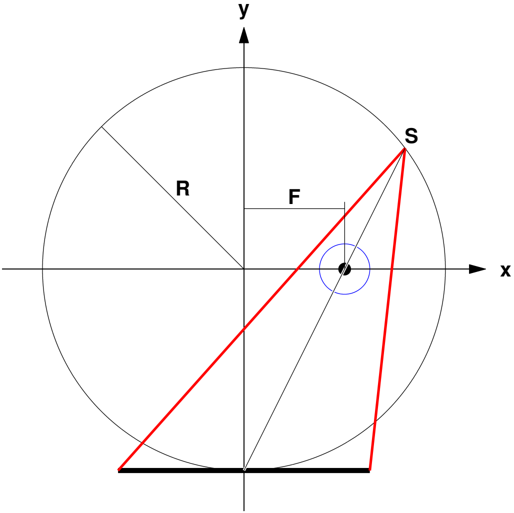
Arc [50, 310]°



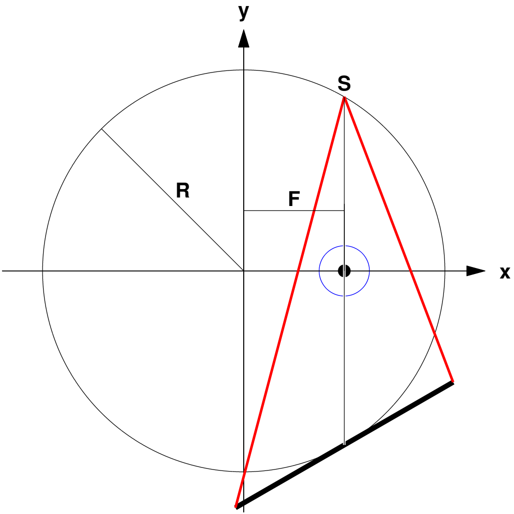
Arc [-130, 130]°



Independent source and detector rotation



Independent source and detector rotation



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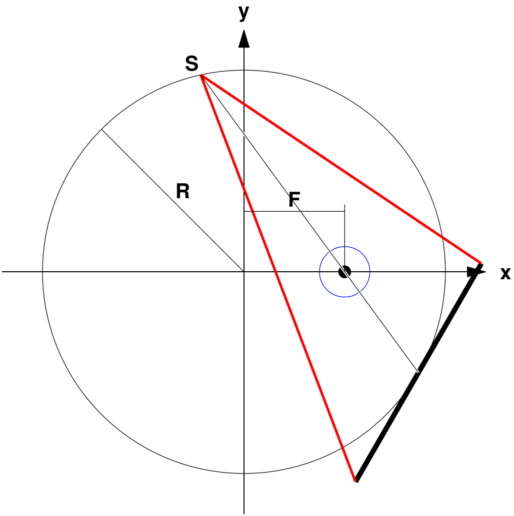
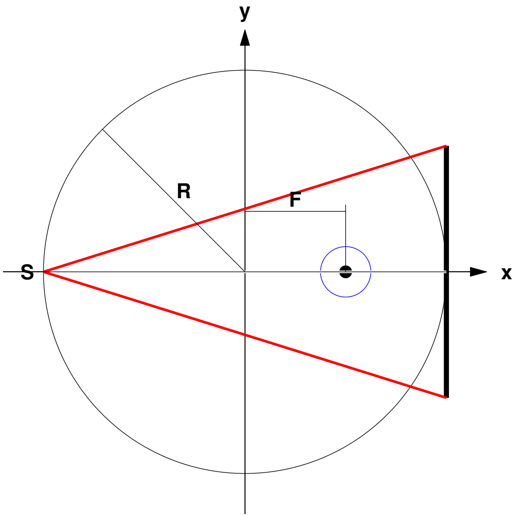
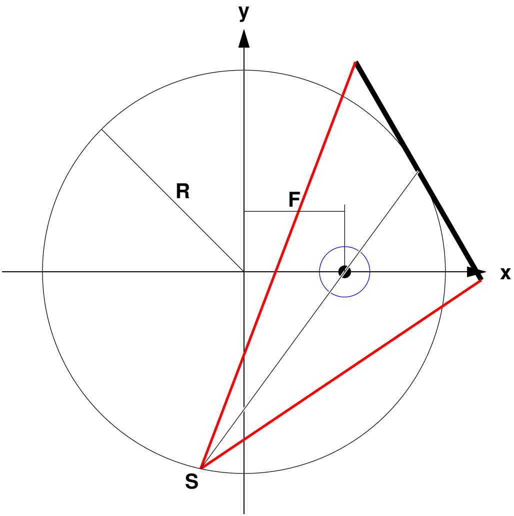


Independent source and detector rotation



Independent source and detector rotation



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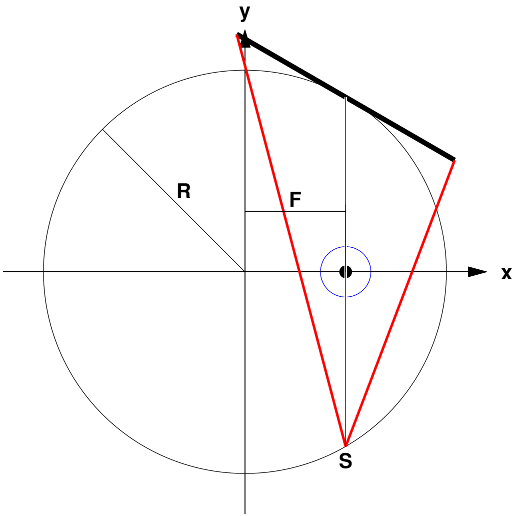

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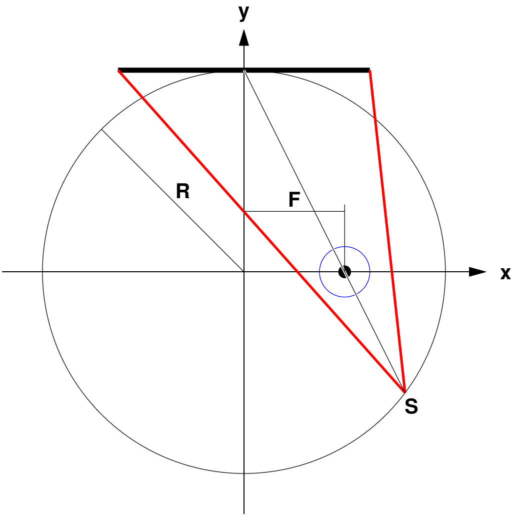

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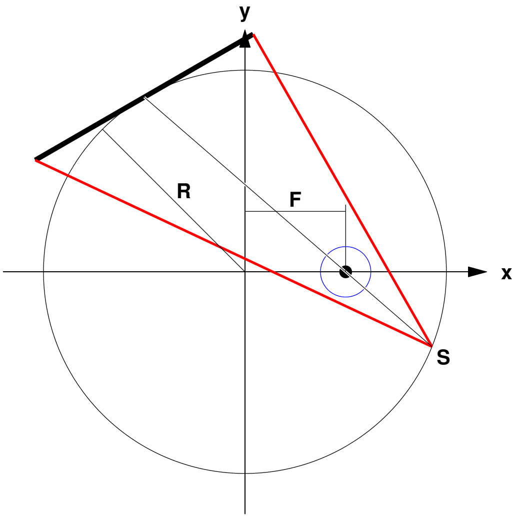

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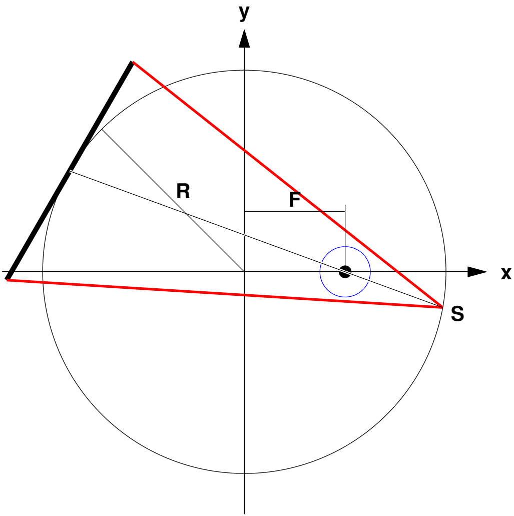

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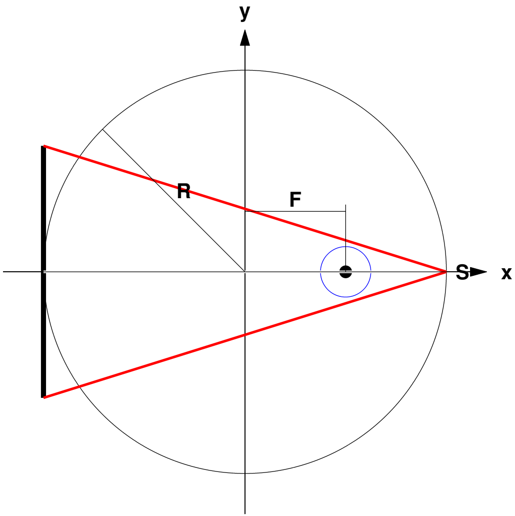

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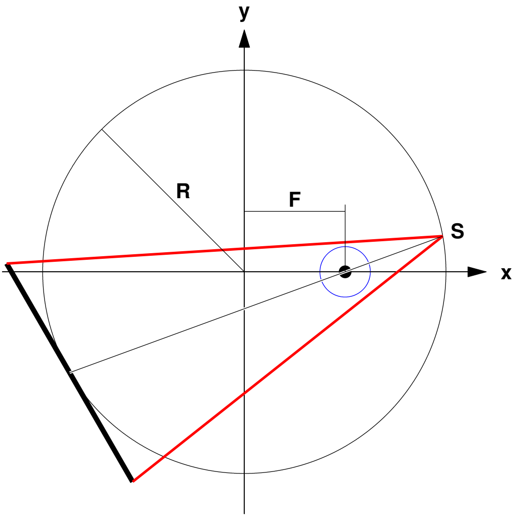

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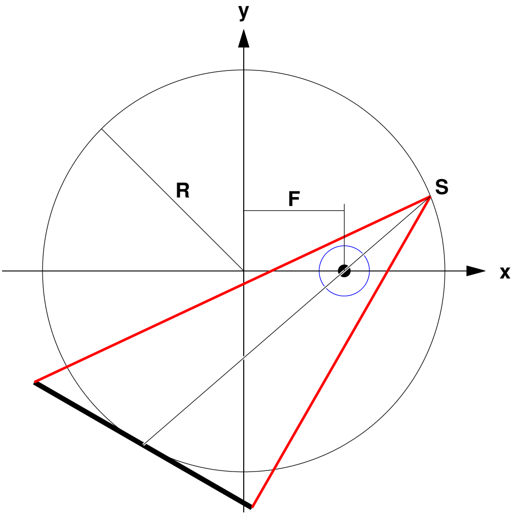

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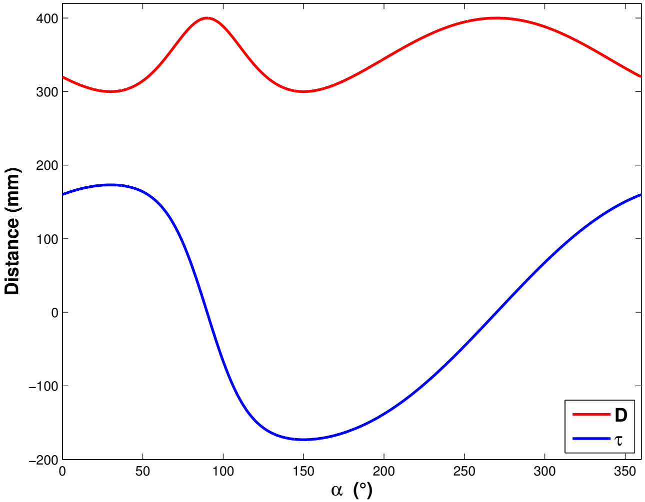
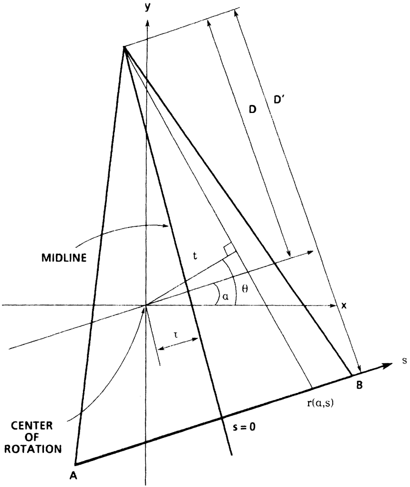


[Crawford *et al.*, Med Phys, 1988]

τ depends on the gantry angle

$$\Rightarrow \tau : \alpha \rightarrow \tau(\alpha) \qquad (20)$$

Note that the exact same derivation has been performed by [Concepcion *et al.*, IEEE TMI, 1992]



Change of variable [Crawford *et al.*, Med Phys, 1988]

Assuming D constant ~~and τ constant~~, the Jacobian matrix is

$$\frac{\mathrm{d}t}{\mathrm{d}s} = (D^2 - \tau s)\frac{Z^3}{D^2} \tag{21}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}s} = \frac{Z^2}{D} \tag{22}$$

$$\frac{\mathrm{d}t}{\mathrm{d}\alpha} = Z\frac{\mathrm{d}\tau}{\mathrm{d}\alpha} \tag{23}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}\alpha} = 1 \tag{24}$$

so its determinant is

$$J = \left|\left(D^2 - \tau s - D\frac{\mathrm{d}\tau}{\mathrm{d}\alpha}\right)\frac{Z^3}{D^2}\right| \tag{25}$$

Change of variable [Crawford *et al.*, Med Phys, 1988]

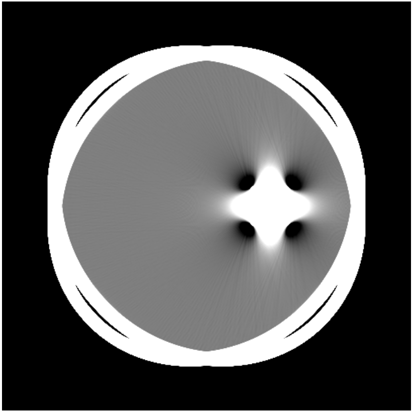
Inversion formula

$$f(r,\phi) = \int_0^{2\pi} \frac{1}{U^2} \int_{-W}^{W} p_f(\alpha, s) \frac{D - \frac{\tau s}{D} - \frac{\mathrm{d}\tau}{\mathrm{d}\alpha}}{\sqrt{s^2 + D^2}} h(s' - s)\, \mathrm{d}s\, \mathrm{d}\alpha \quad (26)$$

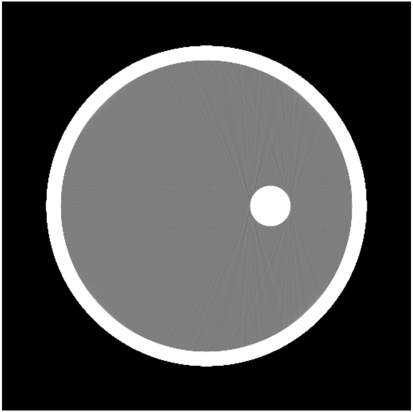
In [Concepcion *et al.*, IEEE TMI, 1992], opposite sign for the new term. Mathematically wrong but experimentally correct (work in progress)...

Experiments [Crawford *et al.*, Med Phys, 1988]

Gray level window: [-582, -482] HU.



Assuming $\tau = 0\,\mathrm{mm}$



Actual $\tau(\alpha) = 3\sin(2\alpha)\ \mathrm{mm}$

Experiments [Crawford *et al.*, Med Phys, 1988]

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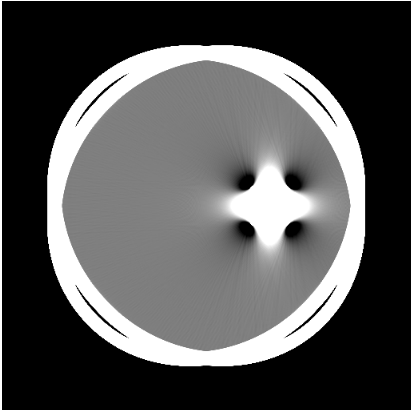

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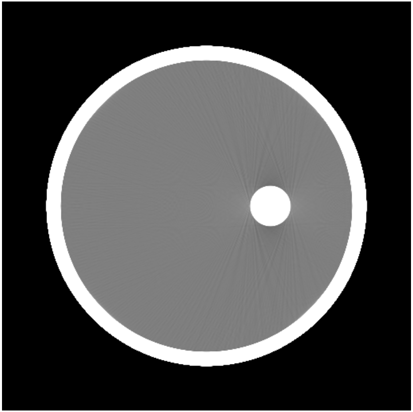
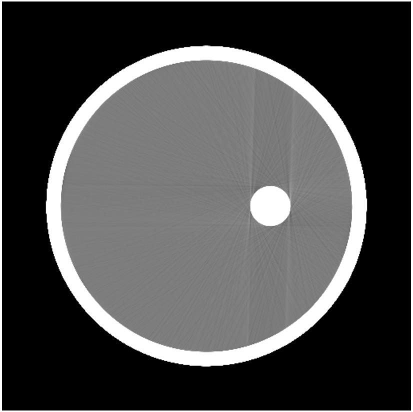


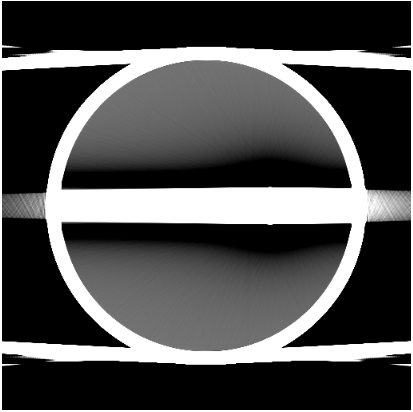
Assuming $\frac{\mathrm{d}\tau}{\mathrm{d}\alpha} = 0\,\mathrm{mm}$

Experiments

Gray level window: [-582, -482] HU.



Actual $\tau(\alpha) =$
$630 \times \mathit{tan}(30^\circ)\,\mathrm{sin}\,(2\alpha)$ mm

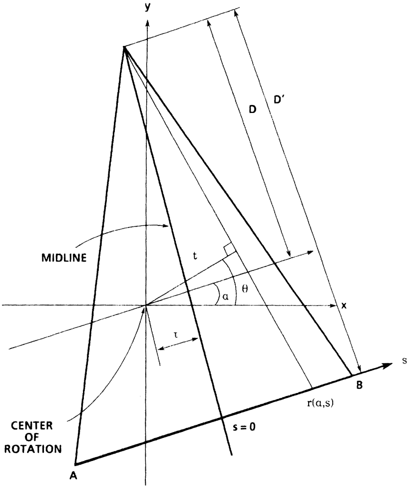


Assuming $\frac{\mathrm{d}\tau}{\mathrm{d}\alpha} = 0\,\mathrm{mm}$

Going further

D also depends on the gantry angle

$$\Rightarrow D : \alpha \rightarrow D(\alpha) \qquad (27)$$



Change of variable

~~Assuming D constant~~ ~~and τ constant~~, the Jacobian matrix is

$$\frac{\mathrm{d}t}{\mathrm{d}s} = (D^2 - \tau s)\frac{Z^3}{D^2} \tag{28}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}s} = \frac{Z^2}{D} \tag{29}$$

$$\frac{\mathrm{d}t}{\mathrm{d}\alpha} = Z\frac{\mathrm{d}\tau}{\mathrm{d}\alpha} + (s+\tau)\frac{Z^3 s^2}{D^3}\frac{\mathrm{d}D}{\mathrm{d}\alpha} \tag{30}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}\alpha} = 1 - \frac{Z^2 s}{D^3}\frac{\mathrm{d}D}{\mathrm{d}\alpha} \tag{31}$$

so its determinant is

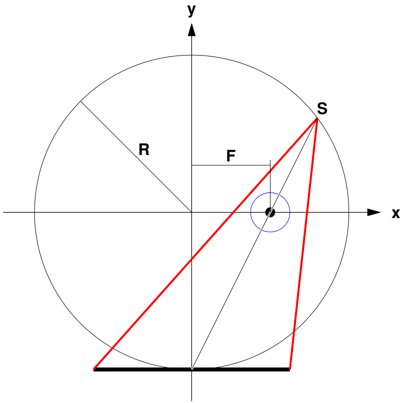
$$J = \left|\left(D^2 - \tau s - D\frac{\mathrm{d}\tau}{\mathrm{d}\alpha} - s\frac{\mathrm{d}D}{\mathrm{d}\alpha}\right)\frac{Z^3}{D^2}\right| \tag{32}$$

Change of variable

Inversion formula

$$f(r,\phi) = \int_0^{2\pi} \frac{1}{U^2} \int_{-W}^{W} p_f(\alpha, s) \frac{D - \frac{\tau s}{D} - \frac{\mathrm{d}\tau}{\mathrm{d}\alpha} - \frac{s}{D}\frac{\mathrm{d}D}{\mathrm{d}\alpha}}{\sqrt{s^2 + D^2}} h(s' - s)\,\mathrm{d}s\,\mathrm{d}\alpha \tag{33}$$

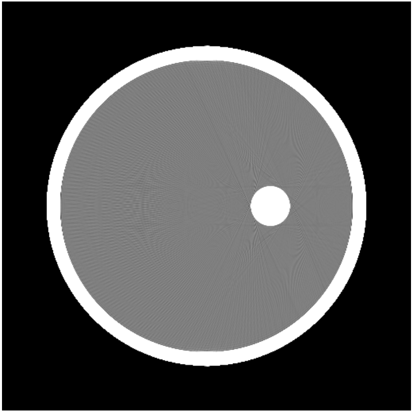
Experiments



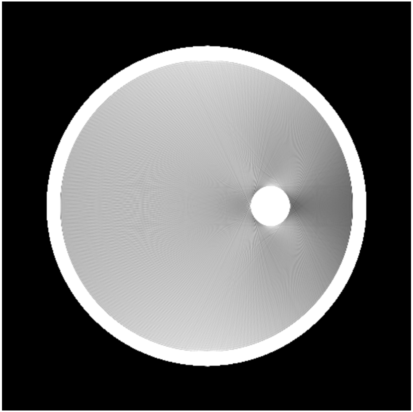
- $R = 200\,\mathrm{mm}$, $F = 100\,\mathrm{mm}$
- Projections
  - 2000
  - 1000 samples
  - $0.25\,\mathrm{mm}$ spacing
- Reconstruction
  - 512×512 pixels
  - $0.125\,\mathrm{mm}^2$ spacing
  - Centered around (F,0)

Experiments

Gray level window: [-582, -482] HU.



New formula



Assuming $\frac{\mathrm{d}D}{\mathrm{d}\alpha} = 0\,\mathrm{mm}$

Conclusions

- Tilting an ideal detector improves CT resolution
- Inversion formula for any 2D derivable trajectory
- Requires the derivative of the parameters with respect to the gantry angle

Open questions

- What is the sensitivity to noise in geometric parameters?
- What is the optimal angular sampling?
- Will it really improve resolution when hardware considerations come into the picture?