

# Dynamic tomography and ROI reconstruction

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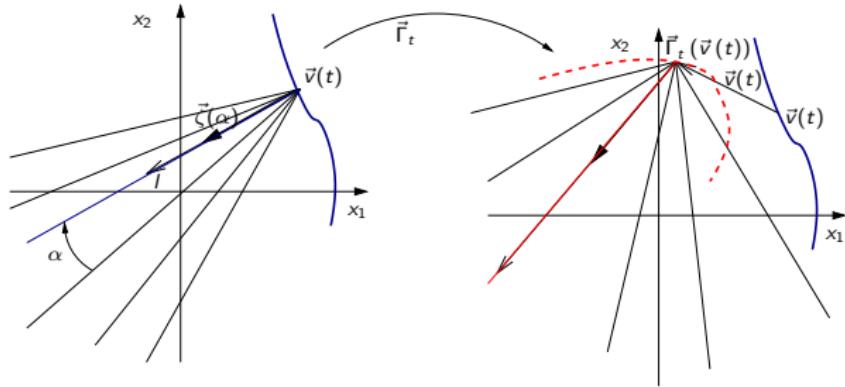
DROITE meeting - Valpré, Lyon- October 25th 2012

# Dynamic tomography: motivation

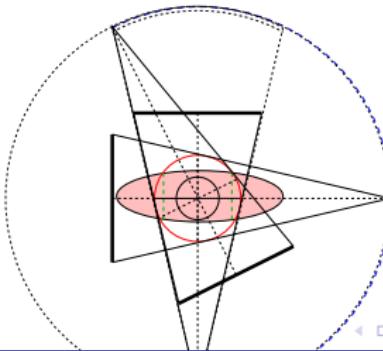
- Cardiac CT, [Li *et al.*, 1995, Grangeat *et al.*, 2002], gating methods [Kachelriess & Kalender, 1998, Kachelriess *et al.*, 2000, Flohr & Ohnesorge, 2001, Gilland *et al.*, 2002]
- Nuclear imaging acquisition (PET or SPECT) with stochastic modeling including movement
- C-arm for interventional imaging [Krimski & et al, 2005]
- Outstanding presentations at CT2012: sessions M1, M3, T1, W1, ...

# Summary

- Analytic reconstruction



- Region Of Interest reconstruction



# Outline

## 1 Introduction and notations

- Dynamic tomography
- Acquisition geometry preservation

## 2 Mass conservation

- Geometry and mass preservation
- Rebinning formula

## 3 Analytic 2D ROI reconstruction from dynamic data

- Introduction
- Numerical experiments

## 4 Discussion

# Divergent geometry, 2D Fan Beam

- $\mu : \mathbb{R}^d \rightarrow \mathbb{R}$  is the (regular) function to be reconstructed ( $d = 2, 3$ )
- $\vec{v} \in \mathbb{R}^d$  denotes a point (an X-ray source position in X-ray CT), and  $\vec{\zeta} \in \mathbb{S}^{d-1}$  a unit vector (in the direction from the source to the detector in X-ray CT),
- the Divergent Beam transform is defined by

$$\mathcal{D}\mu(\vec{v}, \vec{\zeta}) \stackrel{\text{def}}{=} \int_0^{+\infty} \mu(\vec{v} + l\vec{\zeta}) dl$$

- $\mathcal{C} = \{\vec{v}(t), t \in T \subset \mathbb{R}\}$  in  $\mathbb{R}^d$  is the source trajectory.
- 2D fan beam data  $d(t, \alpha) = \mathcal{D}\mu(\vec{v}(t), \vec{\zeta}(\alpha))$  with  $\vec{\zeta}(\alpha) = (\cos \alpha, \sin \alpha)$ ,  $\alpha \in [0, 2\pi[$

# Movement model

- $\mu(t, \vec{x})$  depends on  $t$
- Assumption  $\mu(t, \vec{x})$  behaves like  $\mu(\vec{\Gamma}_t(\vec{x}))$  where  $\vec{\Gamma}_t$  is a time dependent diffeomorphic deformation, i.e. a smooth bijective mapping on the space  $\mathbb{R}^d$ ,  $d \in \{2, 3\}$ :

$$\begin{aligned}\vec{\Gamma}_t : \quad \mathbb{R}^d &\longrightarrow \mathbb{R}^d \\ \vec{x} &\longrightarrow \vec{\Gamma}_t(\vec{x})\end{aligned}.$$

introduced by Crawford et al [Crawford et al., 1996]

- Mass conservation  $\forall \Omega \subset \mathbb{R}^d$

$$\int_{\Omega} \mu(\vec{\Gamma}_t(\vec{x})) |\det J_{\vec{\Gamma}_t}(\vec{x})| d\vec{x} = \int_{\vec{\Gamma}_t(\Omega)} \mu(\vec{y}) d\vec{y}$$

where  $\det J_{\vec{\Gamma}_t}(\vec{x})$  is the determinant of  $J_{\vec{\Gamma}_t}(\vec{x})$  the Jacobian matrix of  $\vec{\Gamma}_t$  at  $\vec{x}$ . We suppose from now that  $\mu(t, \vec{x})$  is in fact

$$\mu_{\vec{\Gamma}_t}(\vec{x}) = \mu(\vec{\Gamma}_t(\vec{x})) |\det J_{\vec{\Gamma}_t}(\vec{x})|$$

# Dynamic Tomography model

- We suppose from now that  $\mu(t, \vec{x})$  is in fact

$$\mu_{\vec{\Gamma}_t}(\vec{x}) = \mu\left(\vec{\Gamma}_t(\vec{x})\right) |\det J_{\vec{\Gamma}_t}(\vec{x})|$$

- We would like to identify  $\mu$  from

$$\mathcal{D}\mu_{\vec{\Gamma}_t}(\vec{v}(t), \vec{\zeta}) = \int_{\mathbb{R}} \mu\left(\vec{\Gamma}_t\left(\vec{v}(t) + I\vec{\zeta}\right)\right) |\det J_{\vec{\Gamma}_t}(\vec{v}(t) + I\vec{\zeta})| dl$$

for some source trajectory  $\vec{v}(t)$ ,  $t \in T$ , line directions  $\vec{\zeta} \in \mathbb{S}^{d-1}$  and  $\vec{\Gamma}_t$  known.

# Acquisition geometry preservation

The beams  $\vec{v}(t) + \mathbb{R}^+ \vec{\zeta}$  (integration half line) at  $t$  are transformed by  $\vec{\Gamma}_t$  into a half lines.

- the source  $\vec{v}(t)$  is transformed into a new source point  $\vec{\Gamma}_t(\vec{v}(t))$
- the line direction  $\vec{\zeta}$  is transformed into a new direction denoted  $\vec{\Sigma}_t(\vec{\zeta})$  where  $\vec{\Sigma}_t$  is a diffeomorphism on the unit sphere (smooth bijection on the sphere):

$$\begin{aligned}\vec{\Sigma}_t : \quad & \mathbb{S}^{d-1} &\longrightarrow & \mathbb{S}^{d-1} \\ \vec{\zeta} & \longrightarrow & \vec{\Sigma}_t(\vec{\zeta})\end{aligned}.$$

Thus a deformation preserving the divergent geometry satisfies

$$\vec{\Gamma}_t \left( \vec{v}(t) + \mathbb{R}^+ \vec{\zeta} \right) = \vec{\Gamma}_t(\vec{v}(t)) + \mathbb{R}^+ \vec{\Sigma}_t(\vec{\zeta}).$$

# Geometry conservation

Modeling the deformation along each half line  $\vec{v}(t) + \mathbb{R}^+ \vec{\zeta}$

$$\begin{aligned}\gamma_{t,\vec{\zeta}} : \quad \mathbb{R}^+ &\longrightarrow \quad \mathbb{R}^+ \\ l &\longrightarrow \quad \gamma_{t,\vec{\zeta}}(l)\end{aligned}.$$

We have,  $\forall l \in \mathbb{R}^+$ ,  $\forall \vec{\zeta} \in \mathbb{S}^{d-1}$ ,

$$\vec{\Gamma}_t \left( \vec{v}(t) + l \vec{\zeta} \right) = \vec{\Gamma}_t(\vec{v}(t)) + \gamma_{t,\vec{\zeta}}(l) \vec{\Sigma}_t(\vec{\zeta}).$$

Clearly for  $\vec{\Gamma}_t$  to be smooth and bijective,  $\gamma_{t,\vec{\zeta}}$  must be a strictly monotonic smooth function and  $\gamma_{t,\vec{\zeta}}(0) = 0$ .  $(\gamma_{t,\vec{\zeta}}, \vec{\Sigma}_t)$  is the change of the spherical variables  $(l, \vec{\zeta}) \in \mathbb{R}^+ \times \mathbb{S}^{d-1}$  centered on  $\vec{v}(t)$  to the spherical variables  $(\gamma_{t,\vec{\zeta}}(l), \vec{\Sigma}_t(\vec{\zeta})) \in \mathbb{R}^+ \times \mathbb{S}^{d-1}$  centered on  $\vec{\Gamma}_t(\vec{v}(t))$ , associated to  $\vec{\Gamma}_t$ .

# Mass conservation in $\mathbb{R}^d$

if

$$\begin{aligned} & \mu_{\vec{\Gamma}_t} \left( \vec{v}(t) + I \vec{\zeta} \right) \\ &= \mu \left( \vec{\Gamma}_t (\vec{v}(t)) + \gamma_{t,\vec{\zeta}}(I) \vec{\Sigma}_t(\vec{\zeta}) \right) |\gamma'_{t,\vec{\zeta}}(I)| |J_{\vec{\Sigma}_t}(\vec{\zeta})| \frac{\gamma_{t,\vec{\zeta}}^{d-1}(I)}{|I|^{d-1}} \end{aligned}$$

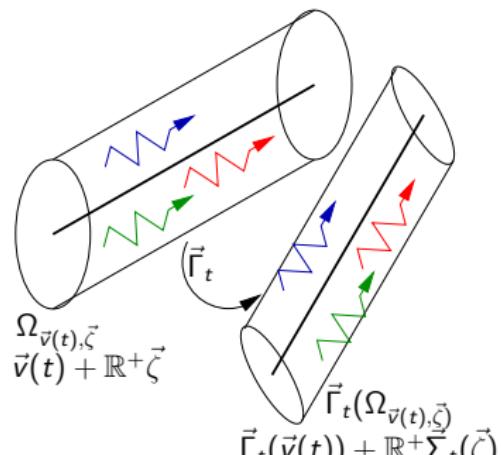
we have ( $\vec{\Gamma}_t$  is a change of variable on  $\mathbb{R}^d$ ).

$$\begin{aligned} & \int_{[l_1, l_2] \times \Omega_S} \mu_{\vec{\Gamma}_t} \left( \vec{v}(t) + I \vec{\zeta} \right) |I|^{d-1} dI d\vec{\zeta} \\ &= \int_{[l_1, l_2] \times \Omega_S} \mu \left( \vec{\Gamma}_t (\vec{v}(t)) + \gamma_{t,\vec{\zeta}}(I) \vec{\Sigma}_t(\vec{\zeta}) \right) \\ & \quad |\gamma'_{t,\vec{\zeta}}(I)| |J_{\vec{\Sigma}_t}(\vec{\zeta})| \frac{\gamma_{t,\vec{\zeta}}^{d-1}(I)}{|I|^{d-1}} |I|^{d-1} dI d\vec{\zeta} \\ &= \int_{[\gamma_{t,\vec{\zeta}}(l_1), \gamma_{t,\vec{\zeta}}(l_2)] \times \vec{\Sigma}_t(\Omega_S)} \mu \left( \vec{\Gamma}_t (\vec{v}(t)) + u \vec{\vartheta} \right) u^{d-1} du d\vec{\vartheta} \end{aligned}$$

# Mass conservation in $\mathbb{R}^d$

the x-ray beam between the source and the detector is some region  $\Omega_{\vec{v}(t), \vec{\zeta}}$  (a “tube”) of approximately constant photon flux,  
 $\vec{\Gamma}_t$  transforms the region  $\Omega_{\vec{v}(t), \vec{\zeta}}$  around the line  $\vec{v}(t) + \mathbb{R}^+ \vec{\zeta}$  into  
 $\vec{\Gamma}_t(\Omega_{\vec{v}(t), \vec{\zeta}})$  around the line  $\vec{\Gamma}_t(\vec{v}(t)) + \mathbb{R}^+ \vec{\Sigma}_t(\vec{\zeta})$ .

$$\begin{aligned} & \Omega_S \int_{\mathbb{R}^+} \mu_{\vec{\Gamma}_t} \left( \vec{v}(t) + I \vec{\zeta} \right) dl \\ & \approx \int_{\Omega_{\vec{v}(t), \vec{\zeta}}} \mu_{\vec{\Gamma}_t} \left( \vec{v}(t) + I \vec{\zeta} \right) I^{d-1} dI d\vec{\zeta} \\ & = \int_{\vec{\Gamma}_t(\Omega_{\vec{v}(t), \vec{\zeta}})} \mu \left( \vec{\Gamma}_t(\vec{v}(t)) + u \vec{\vartheta} \right) u^{d-1} du d\vec{\vartheta} \\ & \approx \mu_{\vec{\Gamma}_t}(\Omega_S) \int_{\mathbb{R}^+} \mu \left( \vec{\Gamma}_t(\vec{v}(t)) + I \vec{\Sigma}_t(\vec{\zeta}) \right) dl \end{aligned}$$



# Mass and geometry conservation, rebinning formula

## Dynamic rebinning formula

If  $\vec{\Gamma}_t$  preserves both the divergent geometry and the mass then

$$\mathcal{D}\mu_{\vec{\Gamma}_t} \left( \vec{v}(t), \vec{\zeta} \right) = c_{t,\vec{\zeta}} \mathcal{D}\mu \left( \vec{\Gamma}_t(\vec{v}(t)), \vec{\Sigma}_t(\vec{\zeta}) \right)$$

# 2D ROI reconstruction with DBP

$H_\alpha \mu$  the Hilbert transform of  $\mu$  in the direction  $\alpha$  is defined by

$$\begin{aligned} H_{\vec{\zeta}(\alpha)} \mu : \mathbb{R}^2 &\longrightarrow \mathbb{R} \\ \vec{x} &\longrightarrow H_{\vec{\zeta}(\alpha)} \mu(\vec{x}) \stackrel{\text{def}}{=} \int_{\mathbb{R}} \mu(\vec{x} - u \vec{\zeta}(\alpha)) h(u) du = \int_{\mathbb{R}} \frac{\mu(\vec{x} - u \vec{\zeta}(\alpha))}{\pi u} du \end{aligned}$$

The backprojection of the derivative of the parallel projections satisfies the equality

$$b_{\alpha, \alpha+\pi}(\vec{x}) \stackrel{\text{def}}{=} \frac{1}{\pi} \int_{\alpha}^{\alpha+\pi} \frac{\partial}{\partial s} p(\phi, \vec{x} \cdot \vec{\theta}) d\phi = 2H_{\vec{\zeta}(\alpha)} \mu(\vec{x})$$

# 2D ROI reconstruction with DBP (FB)

The derivative according to the trajectory parameter  $t$  of the Fan Beam projection is defined

$$g_D(\vec{v}(t), \phi) \stackrel{\text{def}}{=} \frac{\partial}{\partial t} g(\vec{v}(t), \phi)$$

Let  $\overset{<}{b}_{t_1, t_2}$  be the back projection of  $g_D$  on  $[t_1, t_2] \subset T$

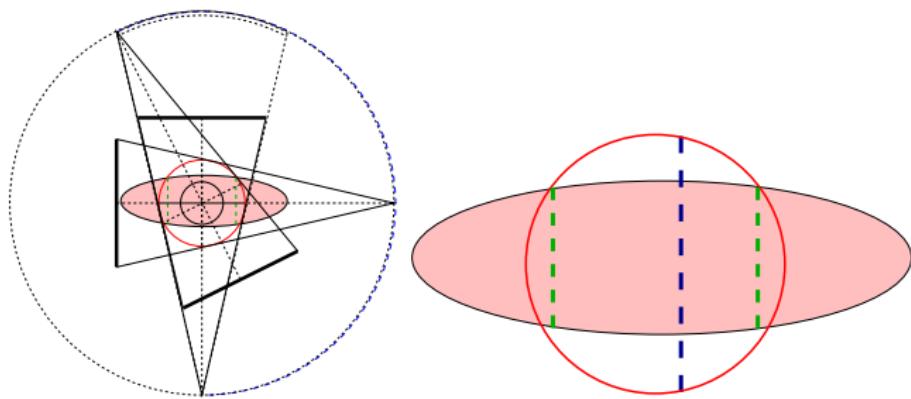
$$\overset{<}{b}_{t_1, t_2}(\vec{x}) \stackrel{\text{def}}{=} \frac{1}{\pi} \int_{t_1}^{t_2} \frac{1}{||\vec{x} - \vec{v}_t||} g_D(\vec{v}(t), \phi) |_{\phi=\arg(\vec{x} - \vec{v}_t)} dt$$

Then [Clackdoyle & Defrise, 2010]

$$\overset{<}{b}_{t_1, t_2}(\vec{x}) = 2H_{\vec{v}_{t_1} - \vec{v}_{t_2}} \mu(\vec{x}) \text{ for } \vec{x} \in [\vec{v}_{t_1}, \vec{v}_{t_2}]$$

# Inversion of the finite Hilbert Transform

Let  $[\vec{y}_1, \vec{y}_2]$  be the intersection of the segment  $[\vec{v}_{t_1}, \vec{v}_{t_2}]$  with a convex containing the support of  $\mu$ . If  $H_{\vec{v}_{t_1} - \vec{v}_{t_2}} \mu(\vec{y})$  is known for  $\vec{y}$  on a segment  $[\vec{y}_{1-}, \vec{y}_{2+}]$  a bit larger than  $[\vec{y}_1, \vec{y}_2]$ , then  $\mu(\vec{x})$  can be reconstructed from  $H_{\vec{v}_{t_1} - \vec{v}_{t_2}} \mu(\vec{y})$  given for  $\forall \vec{y} \in [\vec{y}_{1-}, \vec{y}_{2+}] (\supset [\vec{y}_1, \vec{y}_2] \supset [\vec{v}_{t_1}, \vec{v}_{t_2}])$ .



# Numerical experiments: dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  $\mu_{\vec{\Gamma}_t}$  is dynamically deformed

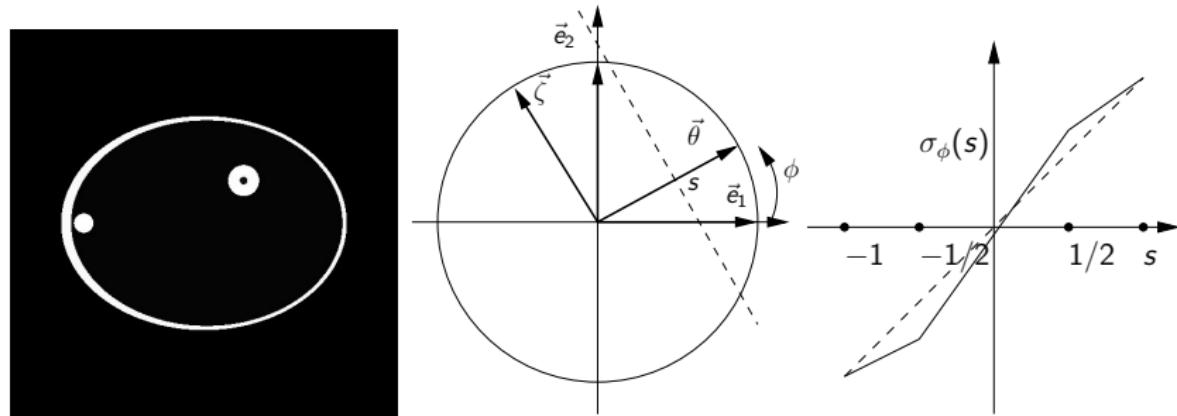


Figure: Left: reference phantom ; right: deformation  $\gamma_\phi$  on the  $s$  axis

420 angular projections, 256 translations  $p(\phi, s) = \int \mu_{\vec{\Gamma}_\phi}(s\vec{\theta} + l\vec{\zeta}) dl$

$$\vec{\Gamma}_\phi \left( s\vec{\theta}(\phi) + l\vec{\zeta}(\phi) \right) = \sigma_\phi(s)\vec{\theta}(\phi) + l\vec{\zeta}(\phi)$$

# Numerical experiments: dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\Gamma_t}$  is dynamically deformed

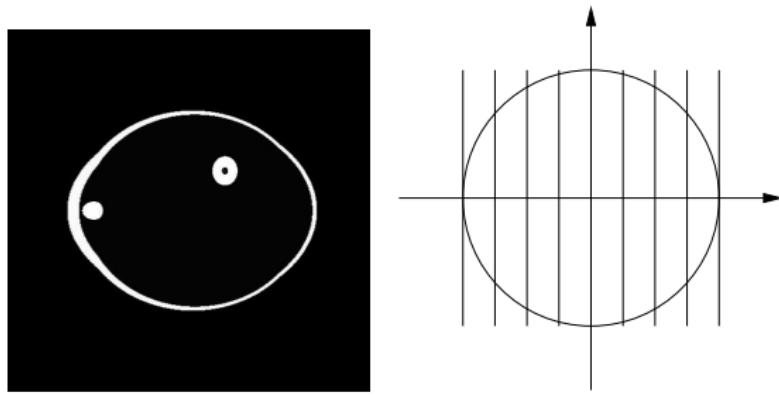


Figure: Dynamic Phantom  $\phi = 0$

# Numerical experiments: dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{\Gamma}_t}$  is dynamically deformed

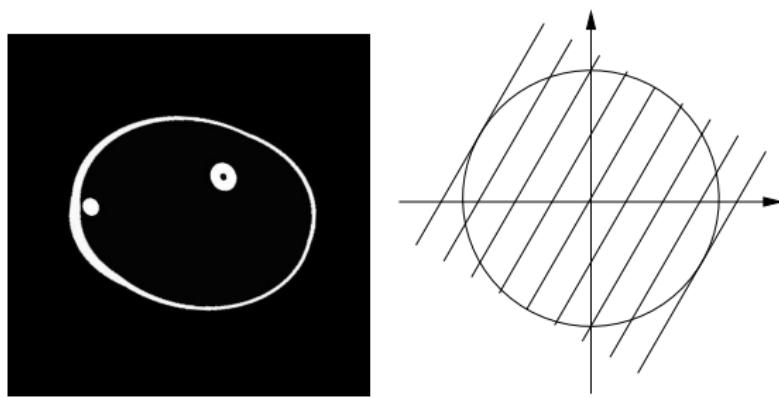


Figure: Dynamic Phantom  $\phi = \frac{\pi}{6}$

# Numerical experiments: dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{\Gamma}_t}$  is dynamically deformed

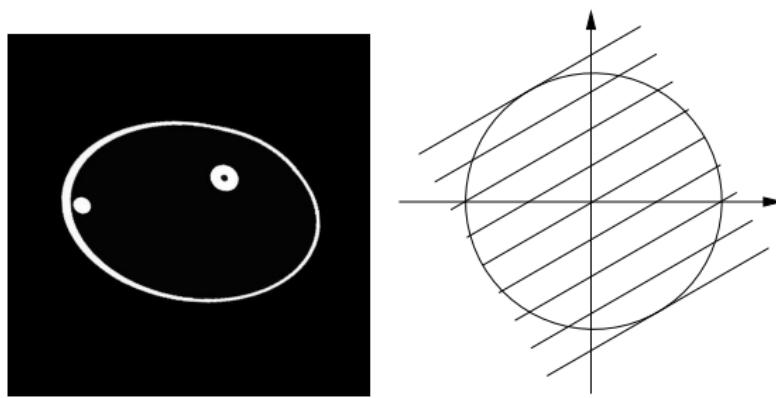


Figure: Dynamic Phantom  $\phi = \frac{\pi}{3}$

# Numerical experiments: dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{\Gamma}_t}$  is dynamically deformed

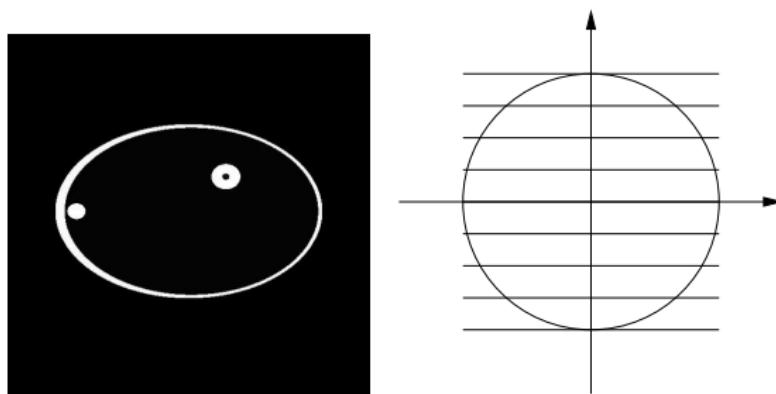


Figure: Dynamic Phanto  $\phi = \frac{\pi}{2}$

# Numerical experiments : dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{\Gamma}_t}$  is dynamically deformed

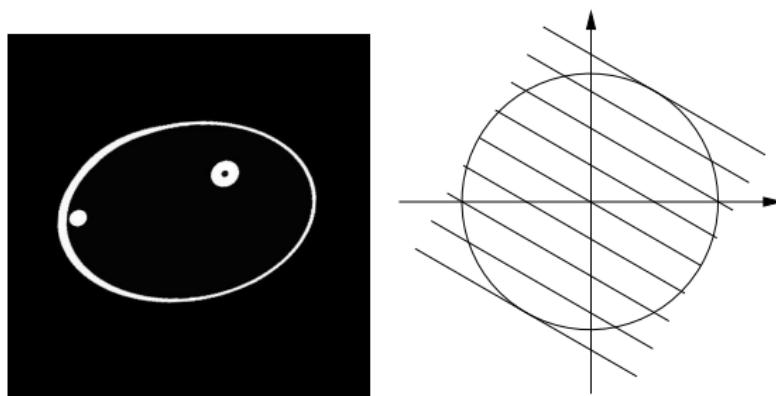


Figure: Dynamic Phantom  $\phi = 2\frac{\pi}{3}$

# Numerical experiments: dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{\Gamma}_t}$  is dynamically deformed

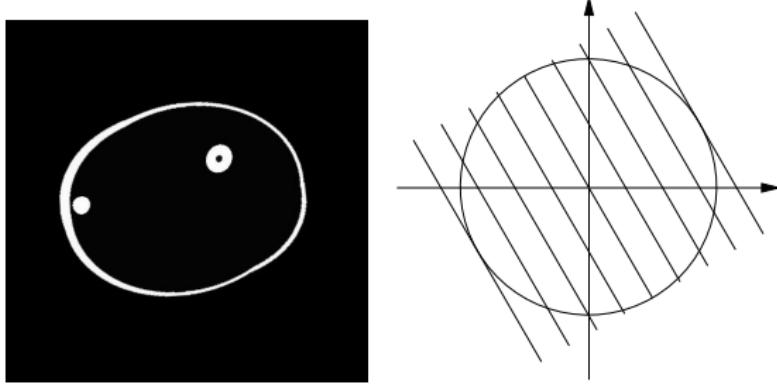


Figure: Dynamic Phantom  $\phi = 5\frac{\pi}{6}$

# Numerical experiments: dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{\Gamma}_t}$  is dynamically deformed

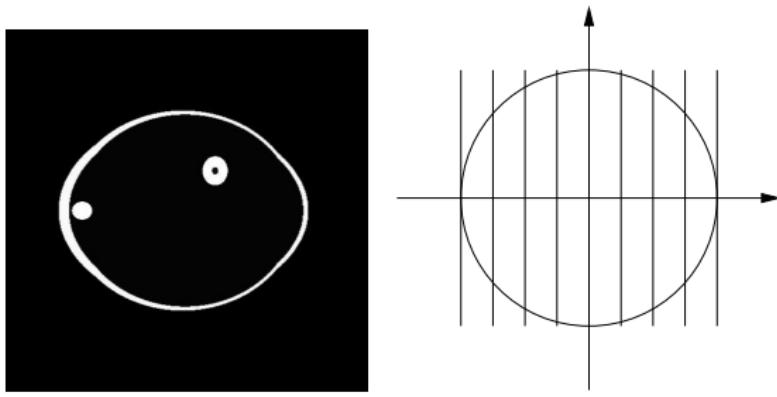


Figure: Dynamic Phantom  $\phi = \pi - \Delta\phi$

# DBP reconstructions from full scann

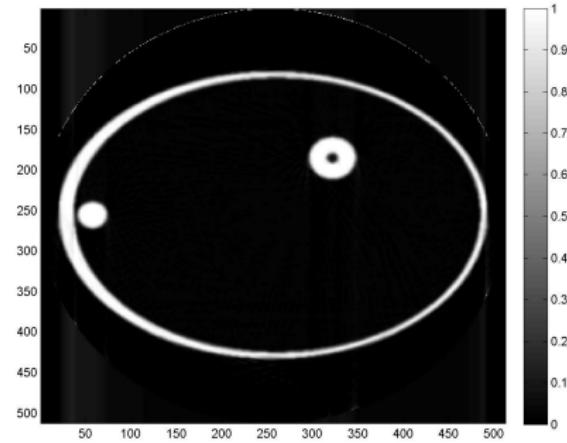
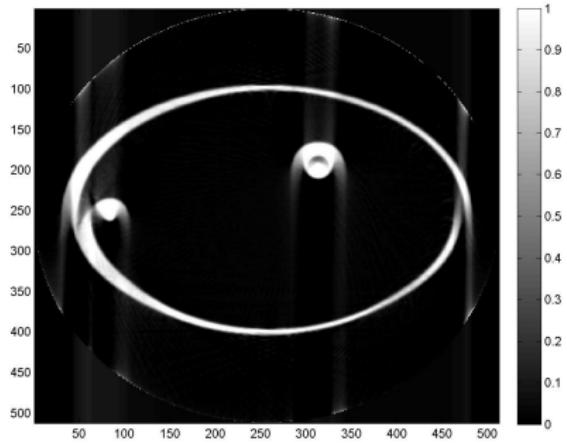
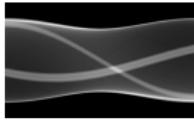
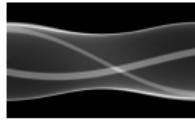


Figure: Left: NO compensation ; Right: Compensation (rebinning)



# DBP reconstructions from truncated projections

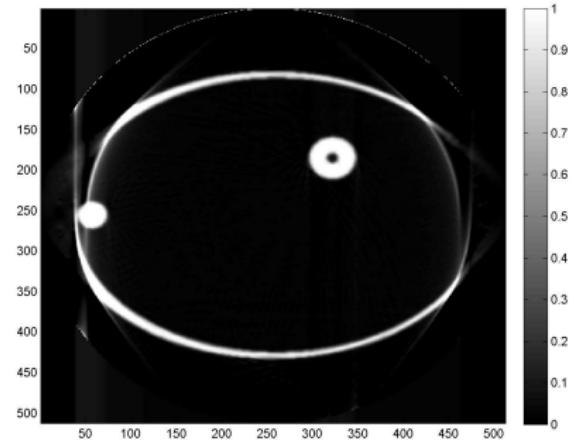
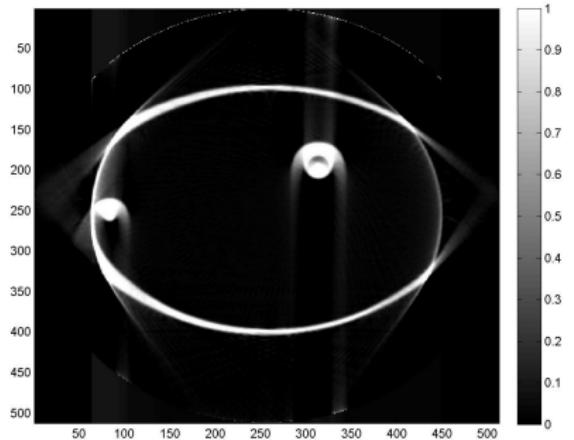
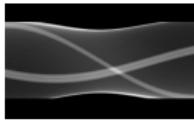
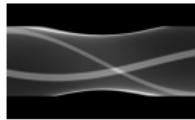
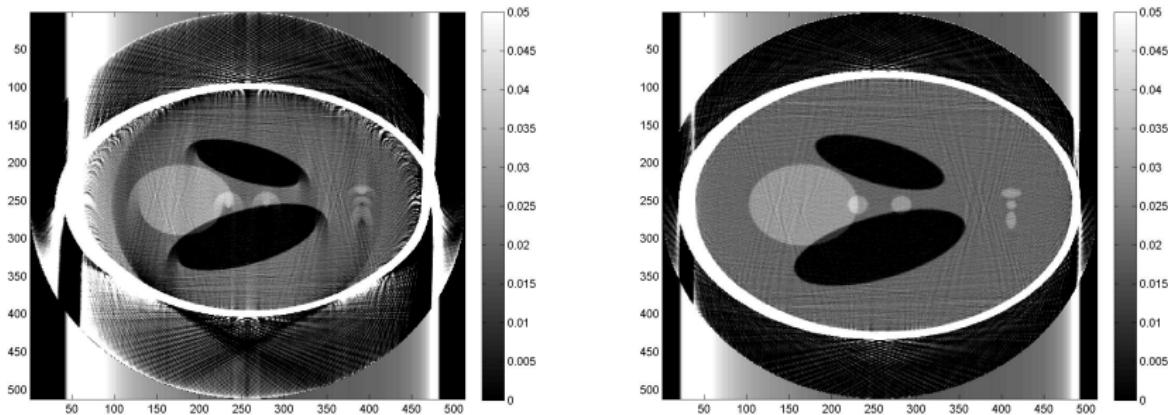


Figure: Left: NO compensation ; Right: Compensation (rebinning)



# SL: DBP reconstructions from full parallel scann

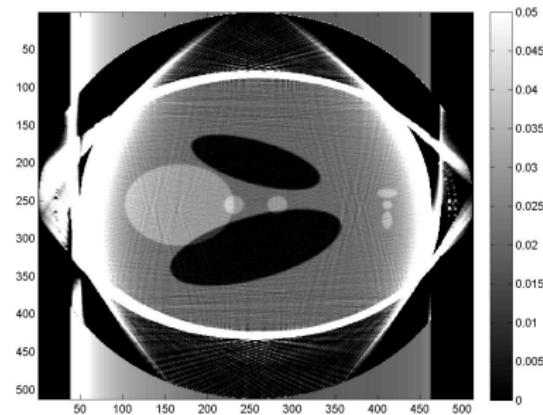
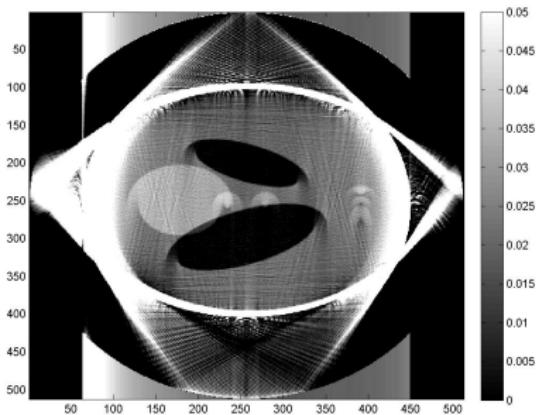


Left: NO compensation ; Right: Compensation (rebinning)

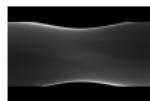
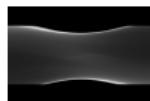


768 angular projections, 512 translations

# DBP reconstructions from truncated projections



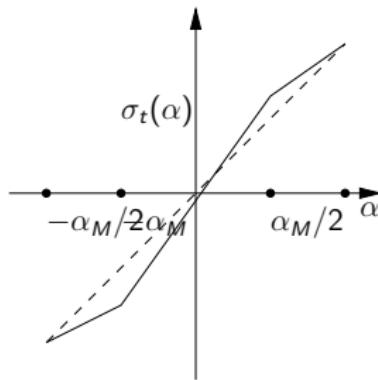
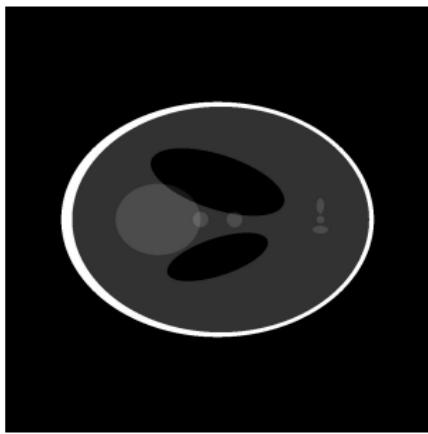
Left: NO compensation ; Right: Compensation (rebinning)



768 angular projections, 512 translations

# Numerical experiments: dynamic phantom

The phantom  $\mu$  is a Shepp and Logan pixel image (linear combination of pixel indicators).



**Figure:** Reference phantom ; right: deformation  $\gamma_t$  on the  $\alpha$  axis

$$\int \mu_{\vec{\Gamma}_t} \left( \vec{v}(t) + l \vec{\zeta} \right) dl$$

$$\vec{\Gamma}_t \left( \vec{v}(t) + l \vec{\zeta}(\alpha) \right) = \vec{v}(t) + l \vec{\zeta}(\sigma_t(\alpha)) \text{ with } \vec{\Sigma}_t(\vec{\zeta}(\alpha)) = \vec{\zeta}(\sigma_t(\alpha))$$

# Numerical experiments: FB dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{F}_t}$  is dynamically deformed

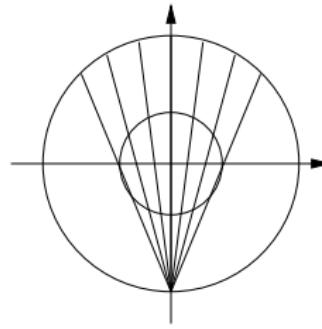
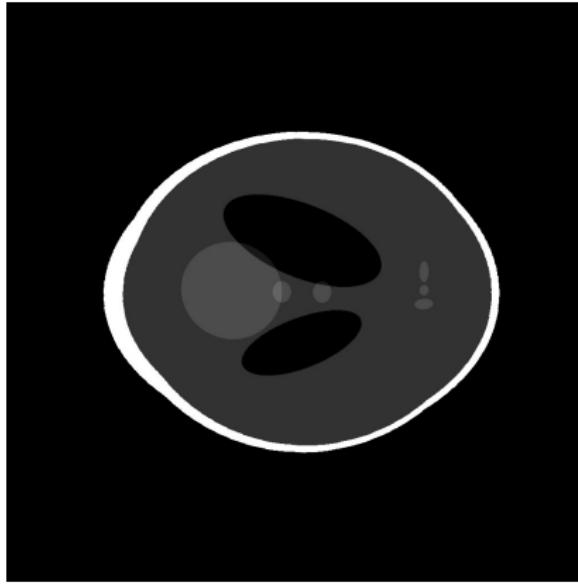


Figure: Dynamic Phantom FB  $\phi = 0$

# Numerical experiments: FB dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{F}_t}$  is dynamically deformed

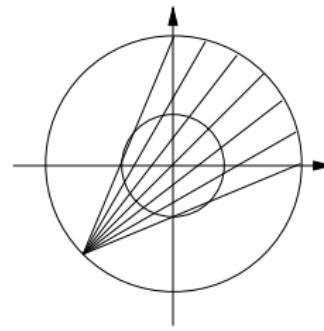
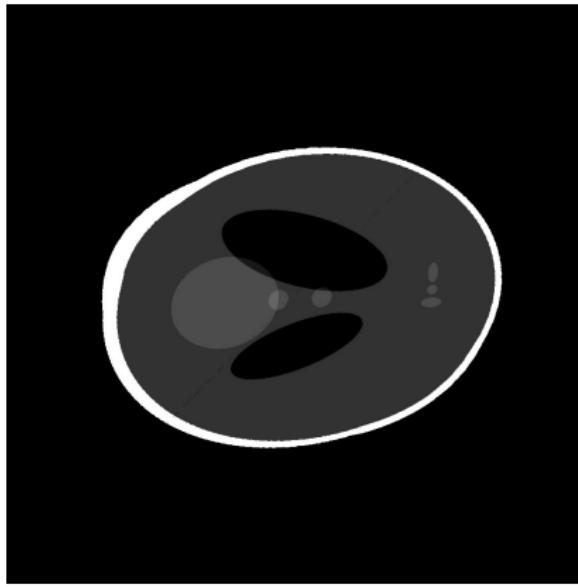


Figure: Dynamic Phantom FB  $\phi = \frac{\pi}{4}$

# Numerical experiments: FB dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{F}_t}$  is dynamically deformed

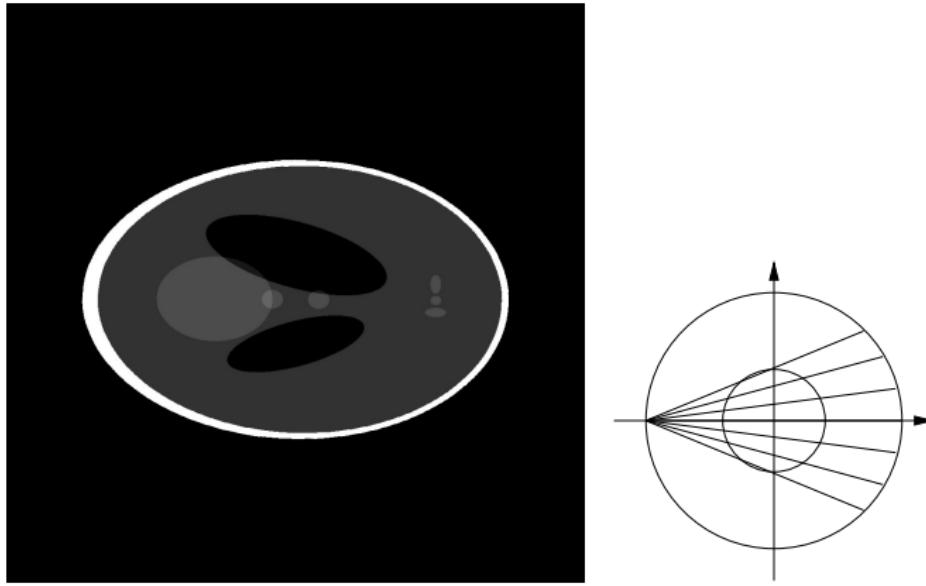


Figure: Dynamic Phantom FB  $\phi = \frac{\pi}{2}$

# Numerical experiments: FB dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{F}_t}$  is dynamically deformed

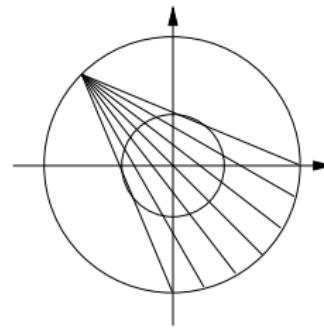
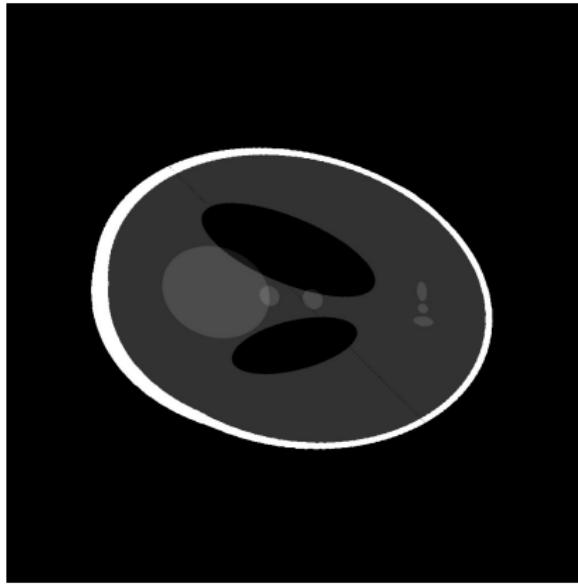


Figure: Dynamic Phantom FB  $\phi = \frac{3\pi}{4}$

# Numerical experiments: FB dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{F}_t}$  is dynamically deformed

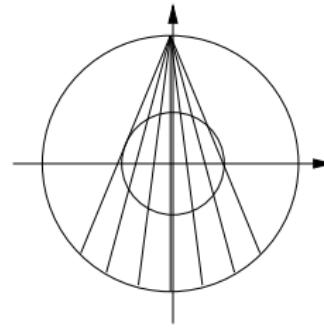
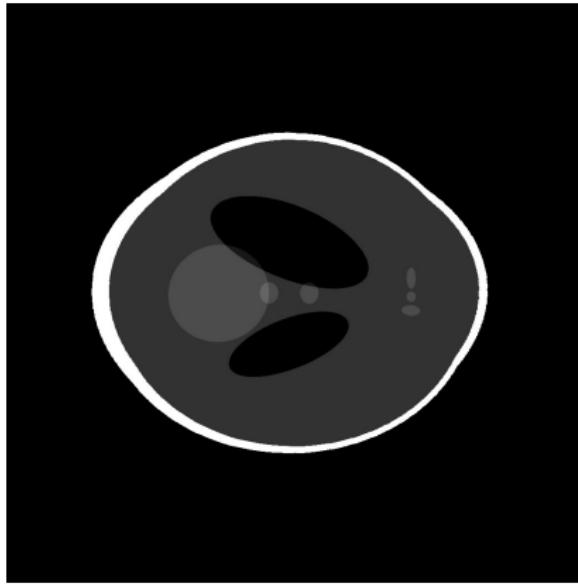


Figure: Dynamic Phantom FB  $\phi = \pi$

# Numerical experiments: FB dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{F}_t}$  is dynamically deformed

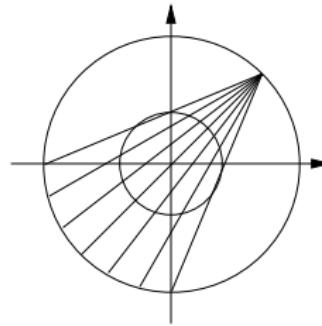
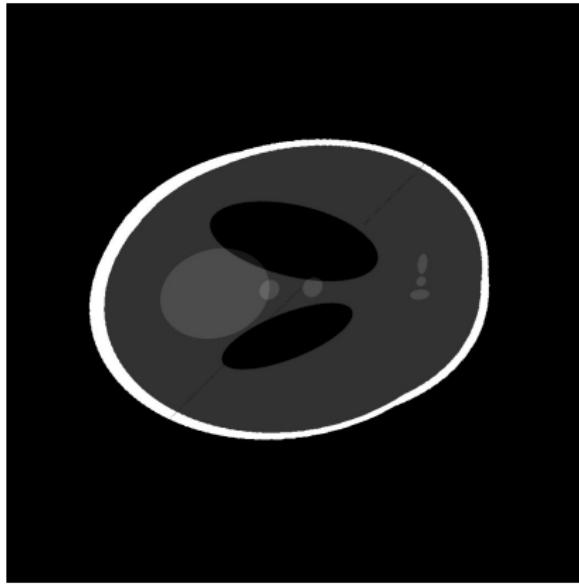


Figure: Dynamic Phantom FB  $\phi = \frac{5\pi}{4}$

# Numerical experiments: FB dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{F}_t}$  is dynamically deformed

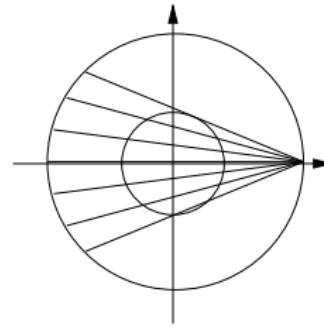
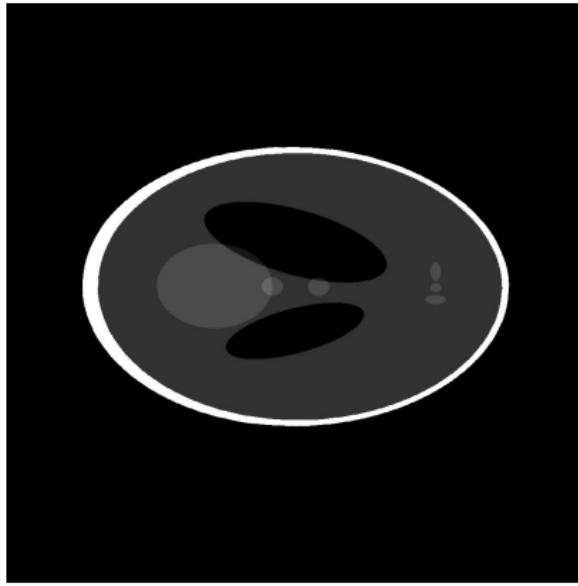


Figure: Dynamic Phantom FB  $\phi = \frac{3\pi}{2}$

# Numerical experiments: FB dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{F}_t}$  is dynamically deformed

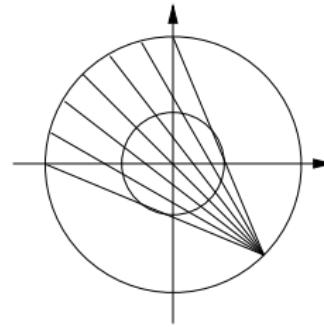
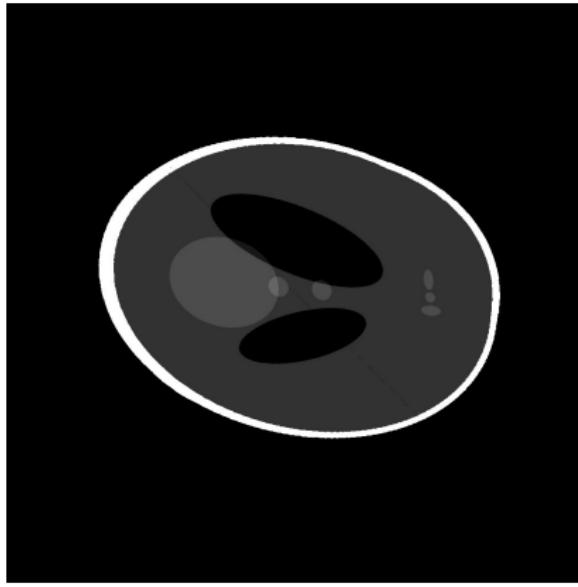


Figure: Dynamic Phantom FB  $\phi = \frac{7\pi}{4}$

# Numerical experiments: FB dynamic phantom

The phantom  $\mu$  is an image (linear combination of pixel indicators).  
 $\mu_{\tilde{F}_t}$  is dynamically deformed

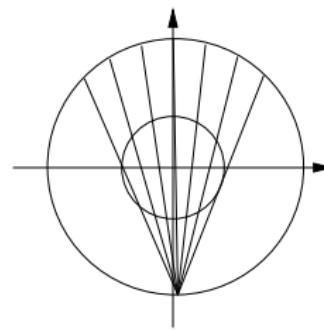
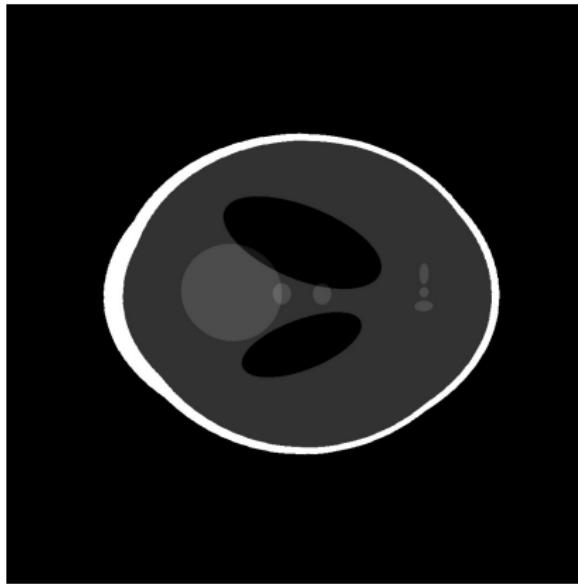


Figure: Dynamic Phantom FB  $\phi = 2\pi - \Delta\phi$

## SL: DBP reconstructions from full FB scann

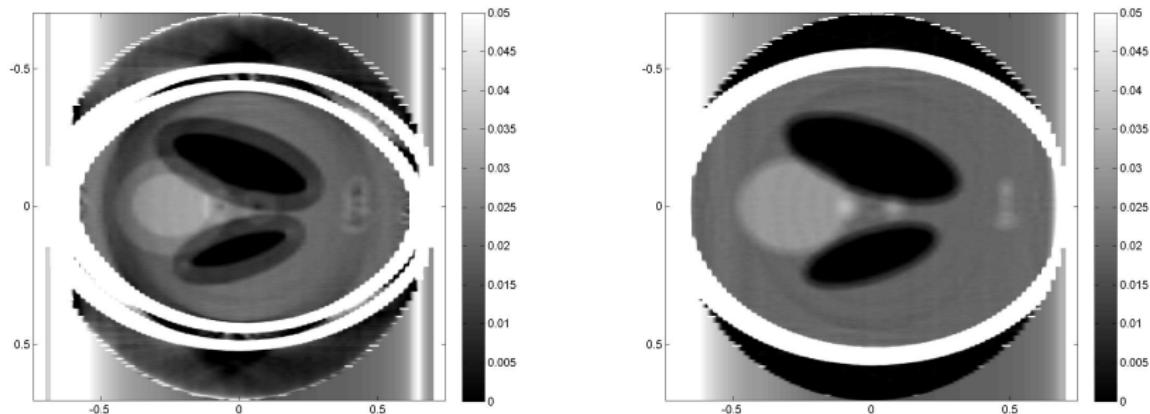


Figure: Left: NO compensation ; Right: Compensation (rebinning)



1024 angular projections, 1024 translations,  $R = 3$

# DBP reconstructions from FB truncated projections

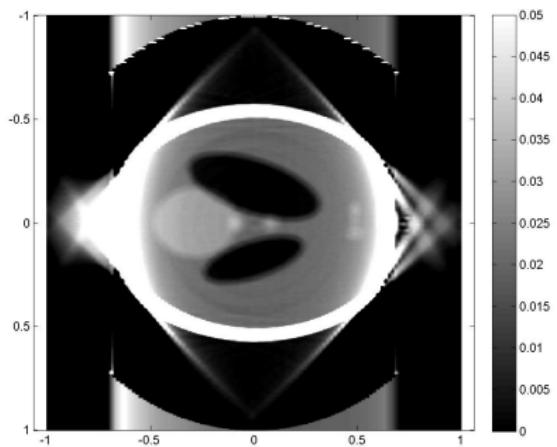
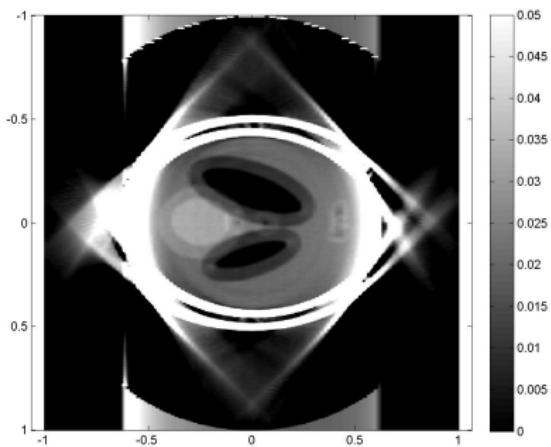
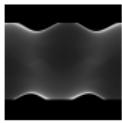
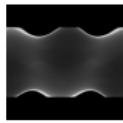


Figure: Left: NO compensation ; Right: Compensation (rebinning)

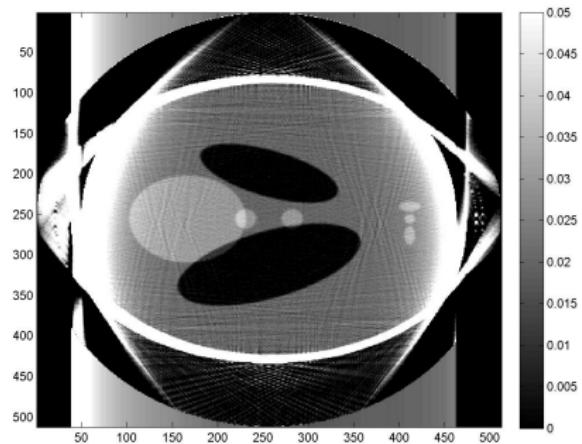
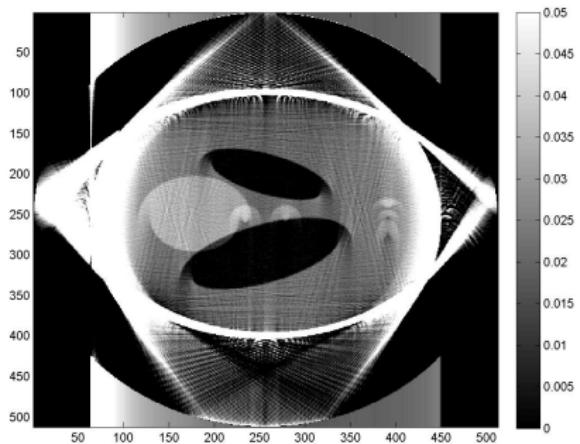


# Summary

- Deformation preserving the acquisition geometry takes care for the Radon Transform.
- Mass preservation takes care for variable changes
- Mass and acquisition line geometry preservation yields a rebinning formula
- ROI reconstruction in the virtual trajectory
- Future work: more numerical experiments + 3D

Thank you! Questions ?

Thank you for your attention!  
Questions ?



# Density, mass and geometry conservation

Because organs are essentially incompressible, we can write the density conservation  $\mu_{\vec{\Gamma}_t}(\vec{x}) = \mu(\vec{\Gamma}_t(\vec{x}))$ . For mass and geometry conservative deformations this yields  $|\gamma'_{t,\vec{\zeta}}(I)| |J_{\vec{\Sigma}_t}(\vec{\zeta})| \frac{\gamma_{t,\vec{\zeta}}^{d-1}(I)}{I^{d-1}} = 1$

## Proposition

*A time dependent deformation preserving the divergent geometry and the mass preserves the density iff*

$$\gamma_{t,\vec{\zeta}}(I) = c_t(\alpha)I \text{ and } \psi'_t(\alpha) = \frac{1}{c_t^2(\alpha)}$$

*with  $c_t(\alpha) > 0$  some  $2\pi$ -periodic smooth function such that*  
 $\int_0^{2\pi} \frac{d\alpha}{c_t^2(\alpha)} = 2\pi$ .

# Line integral model

For line integral model, “mass preserved on the line” if

$$\mu_{\vec{\Gamma}_t} \left( \vec{v}(t) + I \vec{\zeta} \right) = \mu \left( \vec{\Gamma}_t (\vec{v}(t)) + \gamma_{t,\vec{\zeta}}(I) \vec{\Sigma}_t(\vec{\zeta}) \right) |\gamma'_{t,\vec{\zeta}}(I)|$$

then

$$\begin{aligned} \int_{\mathbb{R}^+} \mu_{\vec{\Gamma}_t} \left( \vec{v}(t) + I \vec{\zeta} \right) dI &= \int_{\mathbb{R}^+} \mu \left( \vec{\Gamma}_t (\vec{v}(t)) + \gamma_{t,\vec{\zeta}}(I) \vec{\Sigma}_t(\vec{\zeta}) \right) |\gamma'_{t,\vec{\zeta}}(I)| dI \\ &= \int_{\mathbb{R}^+} \mu \left( \vec{\Gamma}_t (\vec{v}(t)) + I \vec{\Sigma}_t(\vec{\zeta}) \right) dI \end{aligned}$$

thus

$$\mathcal{D}\mu_{\vec{\Gamma}_t} \left( \vec{v}(t), \vec{\zeta} \right) = \mathcal{D}\mu \left( \vec{\Gamma}_t (\vec{v}(t)), \vec{\Sigma}_t(\vec{\zeta}) \right)$$

# Static versus dynamic compensated

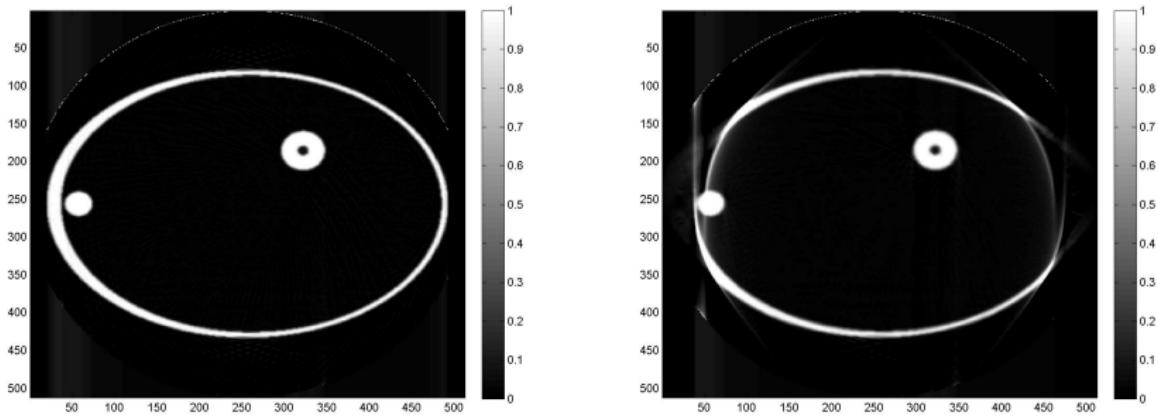


Figure: Left: Static (Full data) DBP ; Right: Dynamic (Truncated data)Corr DBP

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