# Distance-driven binning for proton CT filtered backprojection along most likely paths

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# Introduction

- Proton CT is not new [Cormack, 1963]

- Renewed interest for proton therapy
  - Reduce uncertainty on proton range ($\simeq$ 3% with CT)
  - Lower imaging dose
  - Improved diagnostic / delineation

- Main drawbacks
  - Cost but there are now proton accelerators for treatment
  - Lack of spatial resolution due to Multiple Coulomb Scattering which induces curved proton trajectories

  $\Rightarrow$ Main research effort of the past decade

# pCT reconstruction problem - Physics

- Energy loss via inelastic collisions [Schulte et al., 2005]:

$$-\frac{dE}{dx}(\boldsymbol{x}) = \eta(\boldsymbol{x})S(I(\boldsymbol{x}), E(\boldsymbol{x}))$$

with

$$S(I(\boldsymbol{x}), E(\boldsymbol{x})) = K\frac{1}{\beta^2(E(\boldsymbol{x}))}\left[ln\left(\frac{2m_ec^2}{I(\boldsymbol{x})}\frac{\beta^2(E(\boldsymbol{x}))}{1-\beta^2(E(\boldsymbol{x}))}\right) - \beta^2(E(\boldsymbol{x}))\right]$$

$$\beta(E) = \sqrt{1-\left(\frac{E_0}{E+E_0}\right)^2}$$

- Approximation: $I(\boldsymbol{x}) \simeq I_{water} = 69\,\mathrm{eV},\ \forall \boldsymbol{x} \in \mathbb{R}^3$

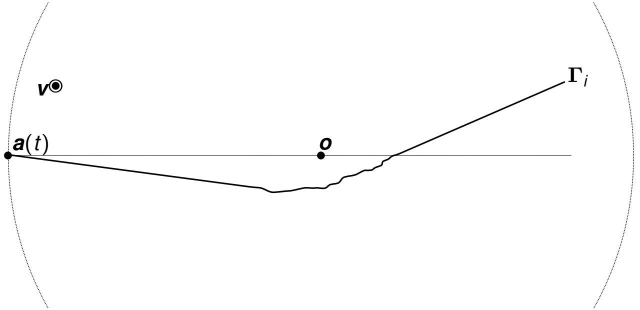
# pCT reconstruction problem - Line integral

$$\int_{\mathbf{\Gamma}_i} \eta(\boldsymbol{x}) d\boldsymbol{x} = \int_{E_i^{out}}^{E_i^{in}} \frac{1}{S(I_{water}, E)} dE = G(E_i^{in}, E_i^{out})$$

with

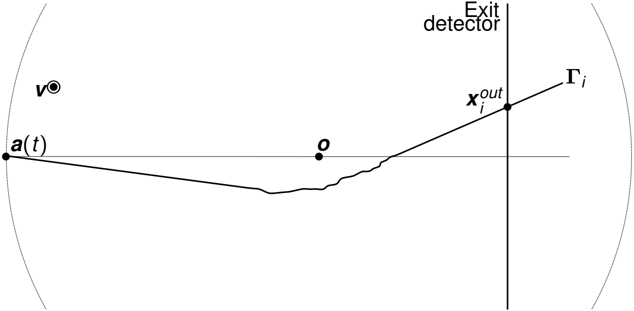
- $i \in \mathbf{I} \subset \mathbb{Z}$ the proton index,
- $\mathbf{\Gamma}_i \in \mathbb{R}^3$ its curved path (measured),
- $\eta : \mathbb{R}^3 \to \mathbb{R}$ the relative electron density (sought),
- $E_i^{out}$ and $E_i^{in}$ the entrance and exit energies (measured).

# Most likely path



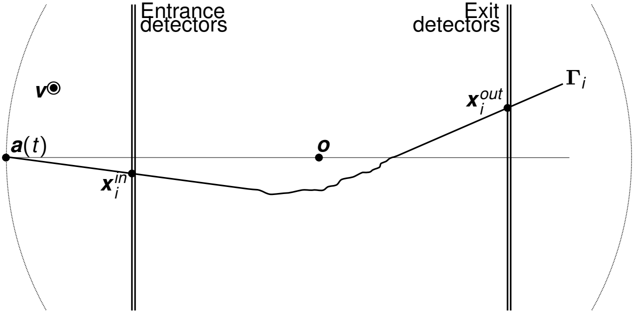

Cone-beam scanner with a circular source trajectory $\boldsymbol{a}(t) \in \mathbb{R}^3$ around the axis defined by the isocenter $\boldsymbol{o} \in \mathbb{R}^3$ and the unit axis $\boldsymbol{v} \in \mathbb{R}^3$.

# Most likely path



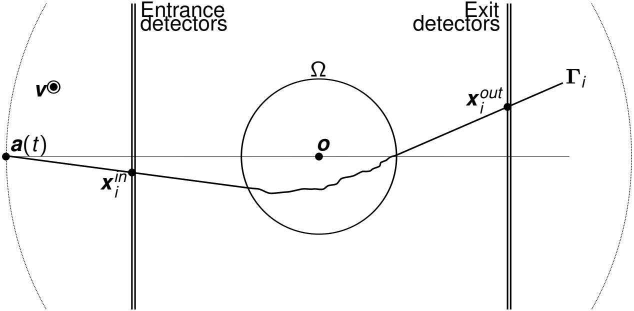

Spatial information on the $i^{\text{th}}$ proton: $\boldsymbol{x}_i^{out}$

# Most likely path



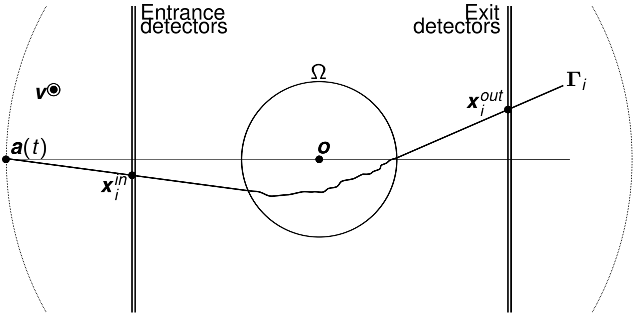

Spatial information on the $i^{\text{th}}$ proton: $\boldsymbol{x}_i^{out}$, $\dot{\boldsymbol{x}}_i^{in}$, $\boldsymbol{x}_i^{in}$, $\dot{\boldsymbol{x}}_i^{out}$

# Most likely path




Spatial information on the $i^{\text{th}}$ proton: $\boldsymbol{x}_i^{out}$, $\dot{\boldsymbol{x}}_i^{in}$, $\boldsymbol{x}_i^{in}$, $\dot{\boldsymbol{x}}_i^{out}$, $\Omega$

# Most likely path

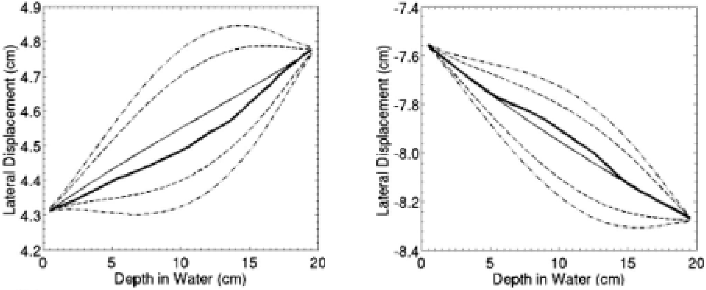



Spatial information on the $i^{\text{th}}$ proton: $\boldsymbol{x}_i^{out}$, $\dot{\boldsymbol{x}}_i^{in}$, $\boldsymbol{x}_i^{in}$, $\dot{\boldsymbol{x}}_i^{out}$, $\Omega$

$\Rightarrow$ Several solutions, [Schulte et al., 2008] used in this work

# Most likely path

[Schulte et al., 2008] for 200 MeV protons



# State-of-the-art of pCT reconstruction

- Filtered-backprojection (FBP) along straight lines
  - Sometimes with smart binning [Penfold, 2010] or cut to eliminate curved paths [Cirrone et al., 2011]

- Iterative reconstruction along most likely paths
  - Algebraic Reconstruction Techniques [Li and Liang, 2004]
  - Statistical, compressed sensing [Penfold, 2010], etc.

$\Rightarrow$ Either FBP along straight lines or iterative reconstruction

# Not true in motion-compensated CT reconstruction

- Non-rigid motion during acquisition yields a reconstruction problem along curved acquisition lines

- Both analytic (approximate) and iterative techniques [Rit et al., 2009]

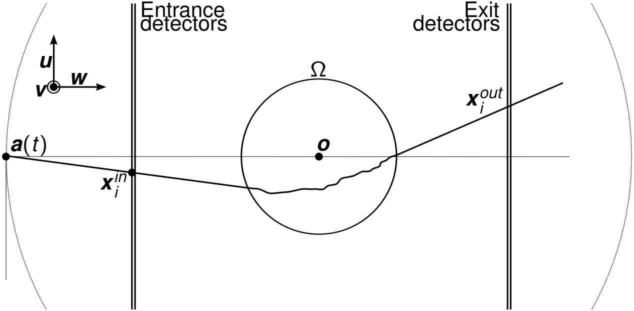
- Difference with protons: list-mode acquisition

# Objective

Approximate FBP algorithm for pCT reconstruction along most likely path

# Rotating coordinate system

- Let $\mathbf{I}_a \subset \mathbf{I}$ be the indices of protons corresponding to the same source position $\boldsymbol{a}(t)$
- $\{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}\}$ is the *rotating coordinate system* where $\boldsymbol{u}$ and $\boldsymbol{w}$ depend on the source position.



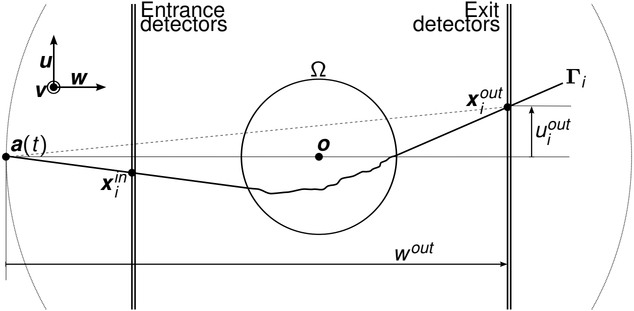
# Grid of pixels for binning

Let $j \in \mathbf{J} \subset \mathbb{Z}^2$ be a set of spatial indices corresponding to a grid of pixels of the exit detector.

$h : \mathbb{R}^2 \to \mathbb{R}$ the indicator of pixel $j$, i.e.,

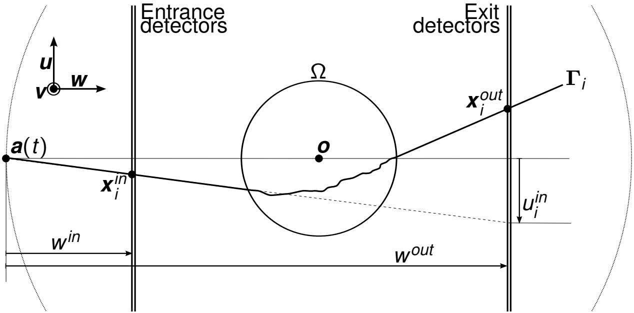
$$h_j(\boldsymbol{y}) = \begin{cases} 1 & \text{if } \boldsymbol{y} \in \mathbb{R}^2 \text{ is in the } j^{\text{th}} \text{ pixel,} \\ 0 & \text{else.} \end{cases}$$

# “Natural” binning



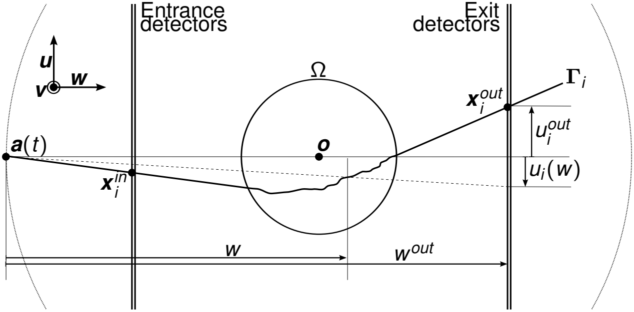

$$g_{j,\boldsymbol{a}}^{out} = \frac{\sum_{i\in \mathbf{I}_a} h_j(u_i^{out}, v_i^{out}) G(E_i^{in}, E_i^{out})}{\sum_i h_j(u_i^{out}, v_i^{out})}$$

# Binning driven by entrance position




$$g_{j,\boldsymbol{a}}^{in} = \frac{\sum_{i \in \mathbf{I}_a} h_j(u_i^{in}, v_i^{in}) G(E_i^{in}, E_i^{out})}{\sum_i h_j(u_i^{in}, v_i^{in})}$$

# Distance-driven binning




$$g_{j,\mathbf{a}}(w) = \frac{\sum_{i \in \mathbf{I_a}} h_j(u_i(w), v_i(w)) G(E_i^{in}, E_i^{out})}{\sum_i h_j(u_i(w), v_i(w))}$$

# Distance-driven binning

- Named from distance-driven (back)projection [De Man and Basu, 2004].

- In practice, binning is computed for several distances $w$ between $w^{in}$ and $w^{out}$.

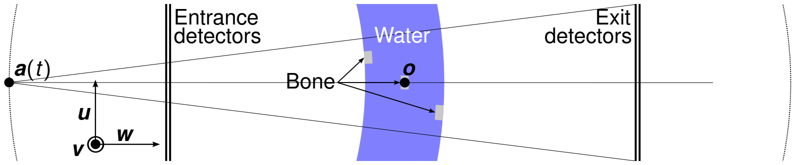
$\Rightarrow$ 4D sinogram $g : \mathbb{R}^3 \times \mathbb{Z} \rightarrow \mathbb{R}$ instead of a standard 3D sinogram, e.g., $g^{out} : \mathbb{R}^2 \times \mathbb{Z} \rightarrow \mathbb{R}$.

- The 3D slice for $w = w^{out}$ is equal to the “natural” binning $g^{out}$.

# Experiments

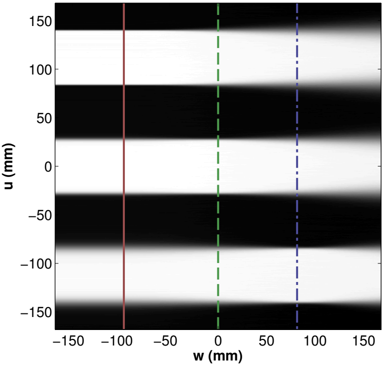
- Monte-Carlo simulations with GATE.
- Ideal pCT scanner
  - $200\,\mathrm{MeV}$ mono-energetic point source placed at $w^s = -1\,\mathrm{m}$
  - Proton characteristics at $w^{in} = -60\,\mathrm{cm}$ and $w^{out} = 60\,\mathrm{cm}$ ($E_i^{in}$, $E_i^{out}$, $\boldsymbol{x}_i^{in}$, $\dot{\boldsymbol{x}}_i^{in}$, $\boldsymbol{x}_i^{out}$ and $\dot{\boldsymbol{x}}_i^{out}$)
- MLP: straight path outside $\Omega$, [Schulte et al., 2008] in $\Omega$
  - Assumes homogeneous object of water

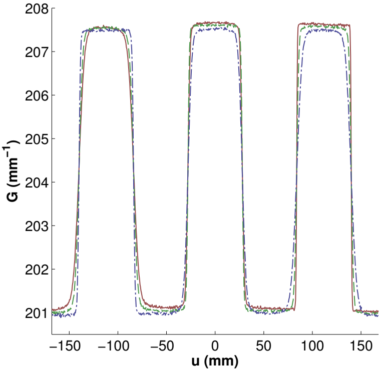
# Experiment #1: one projection only




Spherical object and inserts centered on source
$\Rightarrow$ Projection is a rectangular function if particles follow straight lines (e.g. primary photons)

# Experiment #1: one projection only



# Reconstruction using distance-driven backprojection

If FDK formula is written [Feldkamp et al., 1984]

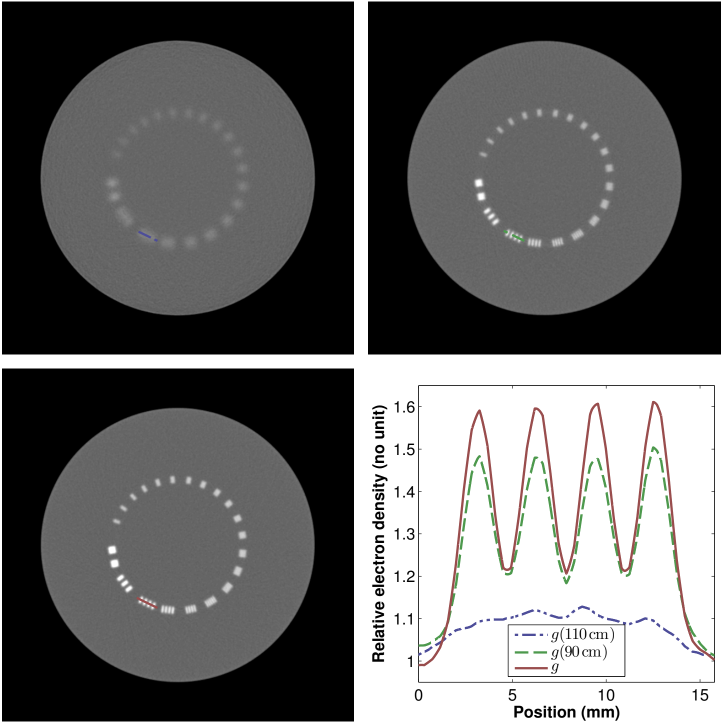
$$\eta(\boldsymbol{x}) = \int_0^{2\pi} \left(\frac{\|\boldsymbol{a}\|_2}{w(\boldsymbol{x})}\right)^2 \tilde{g}_{\boldsymbol{a}}^{out}\left(u(\boldsymbol{x}), v(\boldsymbol{x})\right) d\theta_{\boldsymbol{a}},$$

it becomes,

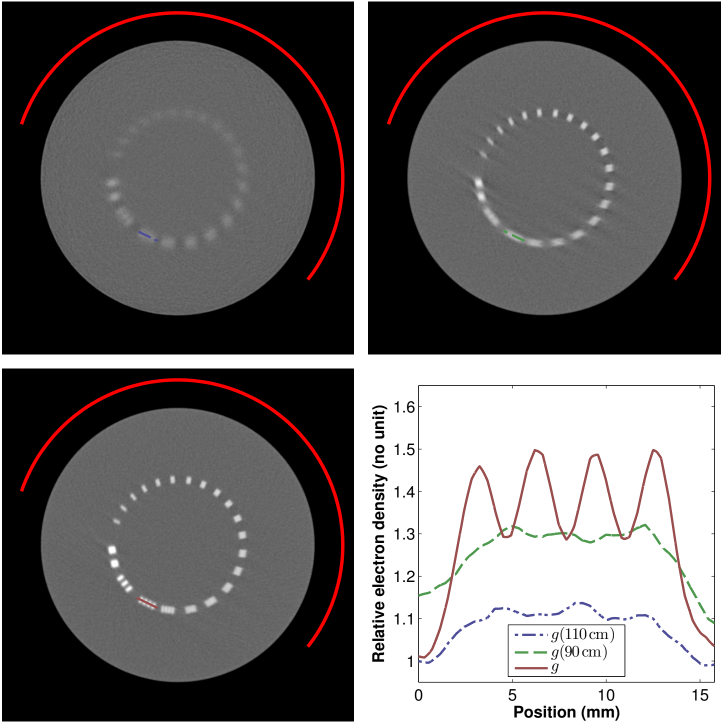
$$\eta(\boldsymbol{x}) = \int_0^{2\pi} \left(\frac{\|\boldsymbol{a}\|_2}{w(\boldsymbol{x})}\right)^2 \tilde{g}_{\boldsymbol{a}}\left(u(\boldsymbol{x}), v(\boldsymbol{x}), w(\boldsymbol{x})\right) d\theta_{\boldsymbol{a}}.$$

$\Rightarrow$ Local compensation as in motion-compensated CT

# Experiment #2: Catphan CTP528



Relative electron density (no unit)
Position (mm)
$g(110\,\mathrm{cm})$
$g(90\,\mathrm{cm})$
$g$



Relative electron density (no unit)
Position (mm)
1
1.1
1.2
1.3
1.4
1.5
1.6
0
5
10
15
$g(110\,\mathrm{cm})$
$g(90\,\mathrm{cm})$
$g$

# Conclusions

- Improved spatial resolution in FBP algorithm using most likely paths by means of distance-driven binning
- No effect on density resolution of homogeneous areas is expected since only high frequencies of projections are modified
  - Being evaluated...
- Use of most-likely paths seems essential for short scans

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