SYNCHROSQUEEZING TO INVESTIGATE CARDIO-RESPIRATORY INTERACTIONS WITHIN SIMULATED VOLUMETRIC SIGNALS

Céline Franco[1,2,3]*, Pierre-Yves Guméry*[1]*, Nicolas Vuillerme*[2]*, Anthony Fleury*[4] *and Julie Fontecave-Jallon*[1]

(1) UJF-Grenoble1/CNRS/TIMC-IMAG UMR5525/PRETA team, Grenoble, F-38041,France
(2) FRE 3405 AGIM Laboratory, CNRS-UJF-UPMF-EPHE, 38706 La Tronche, France
(3) IDS, 2 avenue des puits, 71300 Montceau les Mines, France
(4) Ecoles des Mines de Douai, 941 rue Charles Bourseul, 59500 Douai, France
phone: +33 4 56 52 00 60, email: Celine.Franco@imag.fr

ABSTRACT

The interest of volumetric signals measured by respiratory inductive plethysmography is twofold: (1) clinical as their measurement is non-invasive and (2) physiological as they reveal the interactions between the cardiac and respiratory systems. Interestingly these signals may be seen as the addition of non-stationary cardiac and respiratory modes ordered in frequency. This feature makes them particularly suitable for time-frequency representation based on the pursuit of modes like Synchrosqueezing (SQ). As a preliminary to an application to real cardio-respiratory (CR) signals, a simulated approach has been chosen because of the availability of a reference signal which allows both the validation of the approach and the fine-tuning of the parameters of the algorithm used. Notably, the number of modes necessary to reconstruct the cardiac component from the decomposition of the whole CR signal was analyzed and might be limited to 6.So, this paper aims at investigating the functioning of SQ on one simulated CR signal and comparing its performance with the one of an already tried and tested approach: Empirical Mode Decomposition. In the present set-up, it turns out that the two approaches provide equivalent results.

Index Terms— Synchrosqueezing, Empirical Mode Decomposition, Cardio-respiratory signal, Test bench, Simulation

1. INTRODUCTION

Assessing the cardio-respiratory (CR) interactions in humans is of particular interest from fundamental and clinical physiology points of view. The increasing concern about patient's wellbeing and the advent of wearable devices encourage the development of non-invasive measurements like respiratory inductive plethysmography (RIP) [1]. The CR signal measured by RIP reflects the interaction of the cardiovascular and respiratory systems. It can be modeled as the sum of two components corresponding to the activities of the cardiac and respiratory central oscillators. This yields to a non-stationary signal characterized by a spectral hierarchy of oscillating components with slow amplitude modulation.Thus, it raised the question of achieving a representation able to differentiate these embedded components. Recent methods of data processing differ from the more classical ones by their adaptive and local behavior. Among them, on one hand, empirical mode decomposition (EMD) [2] is particularly interesting. Indeed, as it lies on the extrema of the signal to decompose it into modes *i.e.* components with constant frequency, this method is fully data-driven and needs to predefine neither basis functions nor assumptions about the frequency/scale at stake. However, even if EMD has previously been successfully applied to various physiological signals [3, 4, 5, 6], it suffers from a lack of analytical foundation which limits its understanding and makes it unpredictable. On the other hand, Synchrosqueezing (SQ) [7] combines wavelet analysis and reassignment. It aims at sharpening the time-scale representation by reallocating the coefficients of the continuous wavelet transform (CWT) to another point depending on the local behavior of the representation. However it differs from classical reassignment methods by allowing the exact reconstruction of the signal from the extracted components. Whatever representation is chosen, the knowledge of the spectral hierarchy imposed by the physiological organization of CR signals may be a real advantage to weight the components revealed by the representation against the signal. To make the most of this knowledge, the representation we are looking for is then required to be consistent with this spectral organization proper to CR signal but also limited in the number of modes extracted to make possible their physiological interpretation.

The ability of EMD and SQ algorithms to retrieve a two-tones signal was assessed and compared in [8, 9]respectively resulting in better and more predictable results for SQ. Results showed this ability depends on the ratio of amplitude and frequency between the different components. In a recent study [10], we evaluated the ability of EMD to extract the component of weaker amplitude from simulated CR signals, namely

the cardiac one.

The aim of the present work is twofold: (1) to investigate the functioning of the SQ algorithm in its present state [11] while dealing with simulated stationary CR signals and (2) to assess its performance to reconstruct the cardiac components. In the following, the simulated approach for CR signals generation is first justified and explained. Second, a brief reminder of both SQ and EMD algorithms is provided. Afterwards, the influence of the support of the Fourier Transform of the mother wavelet used in SQ is studied while dealing with simulated CR signals. The number of components needed for reconstruction is also investigated. Finally the performance of the SQ approach is compared with the one of EMD through the calculation of a relative reconstruction error.

2. METHODS

2.1. A simulated approach

The interferential features of CR signals make them both particularly rich and difficult to analyze. To investigate the ability of the SQ method to differentiate the embedded components of CR signals, the benchmark employed in our previous studies [10, 12] was used: (1) a CR signal was generated by adding up a cardiac signal and a respiratory one, each characterized by its frequency (resp. f_A, f_h) and amplitude (resp. a_A, a_h), (2) the SQ transform of this CR signal was calculated, (3) components, with a frequency higher than the cardiac frequency, were extracted and added up to construct an estimated cardiac signal and (4) the simulated and the estimated cardiac signals were then compared through a relative euclidean distance described below (Equ. 3). The model of CR interactions proposed in [13] was used. It takes into account both the mechanical interactions of the ventilatory and the cardiac compartments by modulating the cardiac amplitude by respiration and their respective oscillatory and nonlinear behavior. It yields to a thoracic volume V_{th} modeled as the sum of the alveolar volume V_A and the heart volume V_h. Even if the triangular model used for V_h would be worth improving, a simulated approach seems to be the best and more reliable way to assess objectively the performance of a treatment and to investigate its possibilities. For further details about the model, see [14] and [13]. A CR signal was simulated corresponding to a physiological situation well-handled by EMD [10] that is to say with a ratio between respiratory and cardiac amplitudes ($a = a_A/a_h = 5.95$) and a ratio of frequency ($f = f_A/f_h = 0.17$). In Fig. 1, a part of the simulated signal is illustrated: $f_A = 12$ cycles/minute and $a_A = 1.04$ L for V_A and $f_h = 68.9$ beats/minute and $a_h = 174.6$ mL for V_h. It was sampled at 200 Hz and lasted 35 seconds.

2.2. The synchrosqueezing algorithm

2.2.1. Continuous wavelet transform and reassignment

The algorithm proposed by Brevdo *et al.* [11] was used to calculate the SQ transform of V_{th}. On one hand, the CWT of the signal was calculated using a Bump wavelet ψ whose Fourier transform is denoted by $\hat{\psi}$ (Equ. 1). The central pulsation μ of $\hat{\psi}$ was fixed at 10 rad.s^{-1}. On the other hand, σ corresponds to the half-bandwidth of $\hat{\psi}$ and varies from 1 to 10 with a step of 0.5.

$$\hat{\psi}(\omega) = \exp\left(\frac{-1}{1-(\frac{\omega-\mu}{\sigma})^2}\right) * \mathbb{1}_{|\frac{\omega-\mu}{\sigma}|<1}, \qquad (1)$$

Then, an estimate of the instantaneous frequency (Equ. 2) was calculated from the non-zero coefficients $W_f(a,b)$ of the CWT for the scale a and location b using the formula [15]:

$$\omega_f(a,b) = \begin{cases} \frac{-i}{W_f(a,b)}\frac{\partial}{\partial_b}W_f(a,b) & \text{if } |W_f(a,b)| > 0, \\ +\infty & \text{if } |W_f(a,b)| = 0, \end{cases} \qquad (2)$$

Finally, the SQ transform was obtained by reallocating the instantaneous frequencies according to the following function: $(a,\ b) \rightarrow (\omega(a,b),\ b)$.

2.2.2. Component reconstruction

Afterwards curves were extracted from the SQ transform to reconstruct a part of or the whole signal. However, when extracting curves from the whole time-frequency plane, reconstruction of the cardiac component seems underestimated because of the energy criterion used. To overcome this issue and in accordance with the spectral hierarchy between the cardiac activity and the respiratory one, only frequencies higher than 0.9 Hz were considered. This threshold was chosen towards the cardiac reference at disposal *i.e.* the simulated cardiac signal here.

Then, curves were extracted on the basis of a maximizing energy and minimizing frequency variation criterion. The parameter λ [11] which makes the balance between the energy and the variation parts of the criterion was set by default ($\lambda = 10^3$). To avoid that an abrupt change may cause a dysfunction of the SQ algorithm, the simulation considered in this study was stationary.

Another input to set was the number of components to extract from the SQ transform for reconstruction. Without *a priori* knowledge about it, this number was set from 1 to 30. As we have taken advantage of the knowledge of the signal features thereby working in a particular part of the time-frequency plane, the extracted components were simply added up one by one to reconstruct the cardiac signal from SQ ($V_{h,SQ}$). The combination retained was determined by minimizing the reconstruction error described below (Equ. 3).

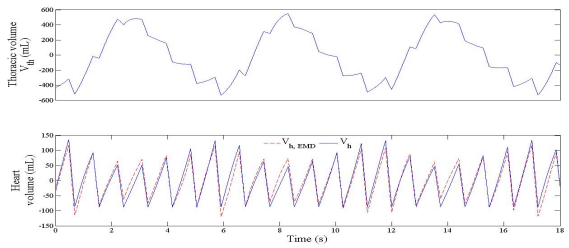

Fig. 1: Reconstruction of the heart volume V_h with EMD $V_{h,\,EMD}$ (bottom) applied to a simulated CR signal V_{th}(top). Only the first 18s are presented for better legibility.

2.3. The Empirical Mode Decomposition algorithm

The simulated CR signal V_{th} was also decomposed with the EMD algorithm [16] into its components, called Intrinsic Mode Functions (IMFs), from the highest to the lowest embedded frequency. It has already been demonstrated that for these values of amplitude and frequency ratio, EMD does constitute a valuable tool to process CR signals [10]. After applying the EMD algorithm, it turns out that only the first IMF had its period corresponding to the cardiac one. The reconstructed cardiac signal from EMD $V_{h,\,EMD}$ was obtained by retaining only the first IMF as illustrated in Fig. 1.

2.4. Estimation of the relative reconstruction error

As a first evaluation, a classical euclidean distance was chosen to quantify the error of reconstruction of the heart volume. The cardiac signal V_h was first divided into cardiac cycles. These cycles were mapped onto the reconstructed cardiac signals $V_{h,\,EMD}$ and $V_{h,\,SQ}$. The relative error was then calculated along each cardiac cycle and consisted in a normalized euclidean distance (Equ. 3).

$$err_j = \frac{\left\| V_{h_j} - V_{h_j,\,EMD\ or\ SQ} \right\|_2}{\left\| V_{h_j} \right\|_2} \text{ for } j = 1 : N, \qquad (3)$$

where N is the number of cycles within V_h and $\|.\|_2$ is the euclidean distance.

3. RESULTS

3.1. Behavior of the SQ algorithm

The results of the SQ algorithm depend on the conjoint resolution in time and frequency of the scalogram imposed by the Heisenberg uncertainty principle. Two behaviors are revealed (Fig. 2): (1) for weak σ (< 4.5), the components extracted are modes of constant frequency, whereas (2) for higher σ (≥ 5), modulations in amplitude and frequency appear, cardiac beats begin to be visualized leading to the extraction of frequency-modulated modes or even entangled frequency modes. A first analysis was performed for $\sigma = 1$ and $\sigma = 5$ to find a common number of components used for reconstruction which minimizes the reconstruction error of the cardiac volume V_h from the reconstructed one $V_{h,SQ}$. The mean, over the 34 cardiac cycles, errors of reconstruction are compared relatively to the number of components used (1 to 30). The reconstruction error has an asymptotic behavior from 6 components (Fig. 3).

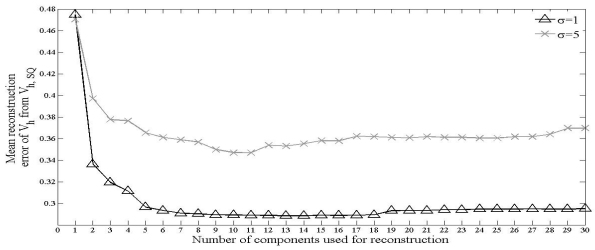

Fig. 3: Error of reconstruction, averaged through the $N = 34$ cardiac cycles, of the simulated cardiac volume V_h from the reconstructed one $V_{h,\,SQ}$ for $\sigma = 1$ (black curve), resp. $\sigma = 5$ (gray curve) depending on the number of components used. An asymptotic behavior is observed from 6 components.

	$\sigma = 1$	$\sigma = 5$
Nb of cardiac cycles N	34	34
Mean $\pm$ std	0.29 ± 0.11	0.36 ± 0.08
Wilcoxon signed ranks test	$Z = -4.12$, $p < 10^{-4}$	

Table 1: Comparison cycle-to-cycle of the relative reconstruction error of $V_{h,\,SQ}$ for $\sigma = 1$ and $\sigma = 5$ with 6 components.

Then, the relative reconstruction error based on 6 components between V_h and $V_{h,\,SQ}$ (Tab. 1) is compared for each cardiac cycle through a Wilcoxon signed ranks test, which test statistic (resp. p-value) is denoted by Z (resp. p), between two reconstructed volumes calculated with a σ leading to each behavior described. Results showed a statistically significant difference between the reconstructed volumes with $\sigma = 1$ and $\sigma = 5$ ($p < 10^{-4}$) in favor of the first one.

3.2. SQ *versus* EMD

	$V_{h,\,SQ}$	$V_{h,\,EMD}$
Nb of cycles N	34	34
Mean $\pm$ std	0.29 ± 0.11	0.30 ± 0.14
Wilcoxon signed ranks test	$Z = -1.51$, $p = 0.13$	

Table 2: Comparison cycle-to-cycle of the relative reconstruction error between $V_{h,\,SQ}$ (for $\sigma = 1$ and 6 components) and $V_{h,\,EMD}$.

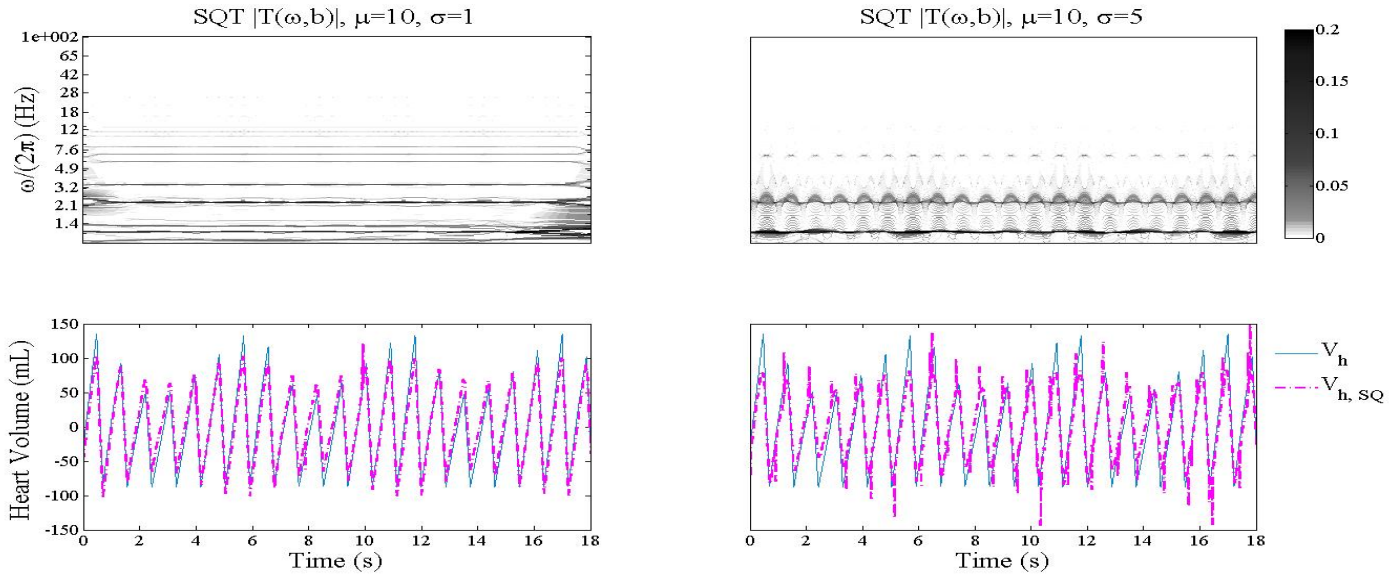

Fig. 2: SQ transform $SQT|T(\omega, b)|$ of V_{th} from a Bump wavelet (top) and reconstruction $V_{h,\ SQ}$ with 6 components of the heart volume V_h (bottom).

The behavior of SQ algorithm while dealing with CR signals depends on the width of the support of $\hat{\psi}$. As it offered the better reconstruction, the parameters of $\hat{\psi}$ were set up to $(\mu = 10,\ \sigma = 1)$. The relative reconstruction errors between V_h and $V_{h,\ SQ}$ (resp. $V_{h,\ EMD}$) are compared for each cardiac cycle through a Wilcoxon signed ranks test (Tab. 2). It results in no statistically significant difference between $V_{h,\ SQ}$ and $V_{h,\ EMD}$.

4. DISCUSSION

Methods such as SQ and EMD based on the search of modes are of great interest to investigate signals made of oscillating modes with slow amplitude modulation like CR signals. This paper aims at investigating the functioning of SQ on such signals under different parametrization and comparing its performance with the one of EMD.

In [9], it was demonstrated that for a two-tones signal, the support of $\hat{\psi}$ influences the ability of SQ to differentiate the embedded components. We proposed to study in what extent this feature is available for CR signals. A simulated approach was chosen to: (1) control the tuning of the parameters of the SQ algorithm and (2) determine whether SQ is a valuable tool for the reconstruction of the cardiac component embedded in a CR signal.The SQ algorithm may be used as it is with small values of σ (< 4.5 rad.s^{-1}) to reconstruct the cardiac volume. Indeed, this range of σ is consistent with the interharmonic distance lower than 1 Hz of the simulated signal.Another input of the present SQ algorithm is the number of components to extract. To set up this parameter, we must bear in mind that our aim is twofold. We are looking for a representation both ordered in frequency to be consistent with the harmonic behavior of CR signals and sharpened enough to make the interpretation of modes and the reconstruction simplified. This requires to get modes as few as possible. The asymptotic behavior of the reconstruction error relatively to the number of components used allows to limit the number of components to 6 (Fig. 3).

On another hand, the performance of SQ has been compared with the one of EMD which had already provided good results on this CR signal[10]. No statistical difference is observed between the two approaches, confirming that SQ may be used on CR signals. Nevertheless, this result needs to be handled with care. The simulation has a ratio of amplitude and frequency favorable to EMD. Moreover a classical euclidean distance was used to assess the performance of the SQ algorithm but the acceptability of a reconstructed cardiac signal depends on the physiological purpose of the cardiac monitoring. In the present set-up, these results give interesting insights into the potential of the SQ approach to deal with CR signals but it would be relevant to insert this clinical dimension by considering a more physiological criterion of assessment as in [1]. Finally, the issue related to the recognition of cardiac modes from respiratory ones was put aside thereby limiting the extraction of components to the part of the time-frequency plane corresponding to the cardiac domain. One purpose of our future works on EMD and SQ is to free ourselves from this constraint to distinguish cardiac from respiratory components without *a priori* hypothesis on the range of frequency and to base the kind of modes recogni-

tion on the spectral hierarchy of modes. Another drawback of SQ in comparison with EMD is its ability to deal with intermittent modes corresponding to possible physiological phenomenons like apnoea, arrhythmia... Indeed, the reconstruction phase of the SQ algorithm only extracts continuous components with significant energy but does not consider abrupt non-stationarities (intermittent extinction of one embedded component, important skip in frequency...). Further works will be dedicated to the development of the reconstruction phase of the SQ algorithm to take into consideration such behaviors. A comparison of EMD (or its noise-assisted variants) and SQ on simulated signals from the whole physiological range of amplitude and frequency and on real signals would also be of great interest and are under process.

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