Towards a suitable time-scale representation of cardio-respiratory signals through Empirical Mode Decomposition algorithms: a simulation and validation tool

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Abstract—To what extent is Empirical Mode Decomposition (EMD) able to differentiate the embedded components of a cardio-respiratory (CR) signal? We intend to answer this question by providing a tool which compares the performances of the original EMD algorithm with those of a noise-assisted version (CEEMD) on simulated CR signals, depending on the frequency and amplitude ratios between their respiratory and cardiac components. A statistical Bland & Altman test checks the matching of stroke volumes calculated from the extracted cardiac components. A dedicated simulation and validation tool is proposed to assess the ability of different EMD algorithms to extract the cardiac volume within the simulated CR signal depending on the frequency and amplitude ratios between the respiratory and cardiac components.

I. INTRODUCTION

This paper addresses the question of a good time-scale representation of physiological signals containing many entangled components. In particular we focus on cardio-respiratory (CR) signals such as volumetric measurements obtained from respiratory inductive plethysmography (RIP). Both the activity of the cardiac and respiratory central oscillators and the nonlinear coupling of the mechanical effects yield to non-stationary signals composed of two added up oscillating modes with slow amplitude modulation. These features prevent a reliable time-scale representation through classical methods. The fully data-driven Empirical Mode Decomposition (EMD) proposed by [1] was successfully used for decomposing numerous kinds of non-linear and non-stationary signals into their different scale components. In the physiological field, EMD and its variants were, for instance, used on postural data [2], lung sound [3], blood pressure [4], respiratory sinus arrhythmia [5], RIP signals [6] and neuronal signals [7]. Despite encouraging results, EMD is not always able to correctly separate all the oscillatory modes from a signal. Indeed, occurrence of intermittency between the present tones leads to “Mode Mixing” and results in the persistence of nonspecific scales [8]. Moreover, it happens that the resolution of EMD is inadequate to detect the presence of distinct tones in some configurations of amplitude and/or frequency ratios. In addition, EMD suffers from a lack of analytical foundation and experiment is the only way to deepen our understanding of its possibilities and limits. That is why, [9] investigated the resolution properties of EMD, onto the sum of two non-linear waveforms and identified the domains within the EMD provides a correct separation of the two individual tones, depending on their frequency and amplitude ratios.

Thus, we work on a simulated thoracic volume modeled as the sum of a cardiac signal and a respiratory one. This study aims at assessing the ability of different EMD algorithms to extract the cardiac volume within the simulated CR signal depending on the frequency and amplitude ratios between the respiratory and cardiac components.

For a brief outline, the second section of this paper is first dedicated to a short description of the CR model and the simulated signals generated. Then the reconstruction of the cardiac component from the decomposition of the simulated CR signals into their underlying oscillatory components by two EMD algorithms is described. In the results section, the stroke volumes (SV) extracted from each reconstructed cardiac signal are statistically compared the theoretical SV, depending on the frequency and the amplitude ratios used for simulation.

II. METHODS

A. Cardio-respiratory model

In this paper, we use the simplified model of CR interactions proposed in [10]. This model mechanically combines a ventilatory compartment and a cardiac one, each defined by its volume. The volume of the rib cage, or thoracic volume $V_{tr}$, is modeled as the sum of the lung volume (or alveolar volume) $V_{A}$ and the heart volume $V_{h}$ (1).

Because of anatomical considerations and position of the heart within the rib cage, cardiac filling and ejection functions are modulated by respiration.

In current practice, the CR model is made of one periodic generator of cardiac volume (cardiac frequency $f_{h}$) and one respiratory module. This latter from [11] is based on mechanical equations and a Lienard oscillator and provides an oscillatory alveolar volume signal (respiratory frequency $f_{A}$). The mechanical cardiac waveform is modeled as a triangle closed to the physiological shape of ventilatory
signals observed in apnea when glottis is closed [12] and modulated in amplitude by the respiration (2).

\[ V_{th}(t) = V_h(t) + V_A(t) = A_h(t) x_v(t; f_h) + a_A V_A(t; f_A) \]  

(1)

\[ A_h(t) = A_h c_1 V_A(t) + c_2 \]  

(2)

The cardiac amplitude \( a_h \) sets the mean SV and the respiratory amplitude \( a_A \) defines the tidal volume. \( c_1, c_2 \) are constants specific to the model. See [10] for a complete description of the model.

### B. Simulations

The model is implemented and simulated under Matlab® using a classical Runge-Kutta integration method. Simulated \( V_{th} \) signals have been generated by varying the heart rate, the breathing frequency, the mean SV and the tidal volume. We define \( f \) as the ratio of respiratory frequency to cardiac frequency \( (f = f_a/f_h) \) and \( a \) as the amplitude ratio between respiratory and cardiac components \( (a = a_A/a_h) \). Considering physiological aspects, the range of \( a \) is \([5-20]\) and the range of \( f \) is \([1/8-1/3]\). We cover the \( \log_{10}(a)-f \) plan with a uniform distribution of 143 physiological stationary simulated situations. The simulated signals are sampled at 50 Hz and last 120 seconds.

In Fig.1, we show an example of simulated signals, corresponding to one physiological situation: \( f_A = 12 \) cycles/minute and \( a_A = 0.5 \) L for the alveolar volume signal \( V_A \) and \( f_h = 52 \) beats/minute and \( a_h = 71.25 \) mL for the heart volume signal \( V_h \).

![Simulated signals](image)

Fig. 1: From top to bottom and limited to 30 seconds, simulated \( V_A, V_h \) and \( V_A \) signals corresponding to the situation with amplitude ratio \( a = 7.02 \) and frequency ratio \( f = 0.23 \).

### C. Empirical Mode Decompositions on simulated signals

The simulated CR signals \( V_{th} \) have then been decomposed into their scaling components through a widespread algorithm EMD [1] and one of its variations: the Complementary Ensemble Empirical Mode Decomposition (CEEMD). Afterwards and for each decomposition, the cardiac volumes \( V_{h,EMD} \) and \( V_{h,CEEMD} \) have been reconstructed by adding the components corresponding to cardiac period.

#### 1) Empirical Mode Decomposition

EMD is an iterative fully data-driven processing method to decompose a signal into its oscillatory components named Intrinsic Modes Functions (IMFs) from the highest embedded frequency to the lowest one. This method is particularly popular because of its local and adaptive features which allow it to tackle non-stationarity and non-linearity respectively. EMD yields to a collection of IMFs characterized by their instantaneous frequency and a residue corresponding to the DC component of the signal. IMFs must verify two properties: 1) zero-local mean 2) a number of zero-crossings and a number of extrema differing at most of one. Adding up all the IMFs and the residue results in the reconstruction of the overall signal.

An example of EMD applied to a simulated \( V_{th} \) is illustrated in Fig.2. To reconstruct the cardiac volume, the respiratory artifacts and the low frequency baseline have to be removed from the overall \( V_{th} \) signal. Throughout simulations, the IMF_{EMD}1 turns out to be the only IMF whose mean period corresponds to the cardiac component. Finally, the cardiac volume reconstructed through EMD denoted \( V_{h,EMD} \) arises from the keeping of the first scale and the deletion of the others

![EMD Decomposition](image)

Fig. 2: From top to bottom, simulated \( V_h \) signal and IMF_{EMD} 1 to 3 of Empirical Model Decomposition applied on \( V_h \).

#### 2) Complementary Ensemble EMD

The ability of EMD to deal with non-linear and non-stationary signals such as biosignals largely contributes to its success. However, a major drawback of the EMD approach, called mode mixing, emerges from experiments [8]. In case of intermittency between the embedded modes of a signal, it happens that a same IMF contains parts of different time scales or that a same mode is fragmented into different IMFs.

To overcome this phenomenon, a noise-assisted method named Ensemble Empirical Mode Decomposition was proposed by [13] and refined by [14] to rise to the CEEMD. These methods are based on the addition of white noise to the original signal \( x \) before performing EMD to make the scales uniformly distributed. For a collection \( (n)_{1\leq n \leq N} \) of \( N \) white noises, EMD is applied to each \( x+n \). For a given scale, the \( N \) resulted IMFs are then averaged to converge towards the true IMF. CEEMD differs from EEMD in the fact that each \( n \) is used twice: once negatively and once positively. This operation lightens the computation load and allows an exact cancellation of the residual noise in the reconstructed
signal. CEEMD was successfully applied to CR signals [6].

CEEMD has been applied to the simulated CR signals \( V_h \) (Fig. 3 for an illustration) with \( N=100 \) white noises and a signal-to-noise ratio of 0.6. For each simulation, due to the CEEMD construction, IMF\(_{CEEMD}^1\) is made of noise. IMF\(_{CEEMD}^4\) and 5 are considered as cardiac because of their mean period consistent with the cardiac information. Further IMFs contain the respiratory component and low frequency baseline. IMF\(_{CEEMD}^2\) and 3 are more doubtful: they have first been rejected and afterwards included one after the other in the sum of IMF\(_{CEEMD}^4\) and 5 to reconstruct a cardiac volume. After all, we have considered the reconstruction of the cardiac volume \( V_{h,CEEMD} \) as the addition of IMFs 2 to 5, since this combination provides the best results according to the criterion of acceptance described below.

**D. Stroke volume estimation and acceptance**

The stroke volume (SV) is the volume of blood ejected by the heart with each beat. From the simulated and the extracted cardiac signals, beat-to-beat SV may be estimated as the amplitude of the cardiac volume signal at each cardiac cycle. For each simulation (120 seconds each), the SV values have then been estimated at each beat for the 3 cardiac signals at disposal: the reference signal \( V_h \), and the two reconstructed signals \( V_{h,EMD} \) and \( V_{h,CEEMD} \). The series are noted \( SV_{ref} \), resp. \( SV_{EMD} \) and \( SV_{CEEMD} \) for \( V_h \), resp. \( V_{h,EMD} \) and \( V_{h,CEEMD} \).

We have analyzed the relations between extracted SV values obtained by decomposition and the reference simulated SV volumes, by Bland & Altman statistical test [15]. For each simulation, the limits of agreement (95% confidence limits) between the SV estimated from reconstructions and the reference are computed. Results are represented in gray scale on the log\(_10\)(a)-f plane, according to the considered simulation. We finally compare the limits of agreement obtained, with the recommendation of [16] for cardiac output measurements, which says that “acceptance of a new technique should rely on limits of agreement (95% confidence limits) of up to \( \pm 30\% \)”.

**III. RESULTS**

Fig.4 illustrates, for the simulation proposed in Fig.1, the superimposition of the simulated cardiac reference signal \( V_h \) and each of the two reconstructed cardiac signals. At first sight, adequacy of the reconstructed cardiac signal from both EMD and CEEMD to the reference signal looks globally acceptable outside strong discrepancies at 16s and 21s in the EMD case. In fact, the limit of agreement for this simulation is 34.2% and 14.8% for EMD and CEEMD respectively and thus leads to a rejection of the EMD method and acceptance of CEEMD (Fig.5).

**Fig. 4: Cardiac signals extracted (bold line) from the previous \( V_h \) signal: \( V_{h,EMD} \) is the IMF\(_{EMD}^1\) on \( V_h \) and \( V_{h,CEEMD} \) is the sum of IMF\(_{CEEMD}^2\) to 5 on \( V_h \). Simulated \( V_h \) signal (dashed line) is shown as reference.**

**Fig. 5: Bland–Altman plot to compare the reference-based \( SV_{ref} \) with the CEEMD-based \( SV_{CEEMD} \) measurements of stroke volume (92 measures over 120 sec.). The gray domain represents the area of acceptance (\( \pm 30\% \)).**

Fig.6 and Fig.7 summarize the Bland & Altman statistical test results for the 143 simulations and for the two
algorithms. The domain of frequency and amplitude ratios within the estimation of SV is satisfactory wider in the case of CEEMD than EMD. So, CEEMD improves the estimation of SV in comparison with EMD. This may result from a better ability of CEEMD to separate a cardiac signal superimposed to a respiratory one. One may note that the boundary of the domain within EMD yields to a perfect separation of the two components of a synthetic two tone signal in the study of [9] in agreement with ours.

Similar Bland & Altman tests have been obtained for the other potential combinations of IMF<sub>CEEMD</sub> described in section II.C. but have resulted in narrower domains of acceptance.

Fig. 6: Limits of agreement (in %) of Bland & Altman statistical tests between estimated SV<sub>EMD</sub> and reference SV<sub>ref</sub> for the 143 simulations. f is the frequency ratio (<f>f/h</f>) and α is the amplitude ratio (<a>α/hα</a>). The white dashed line represents the upper limit of the domain [9] within EMD provides a correct separation of a two tone signal. Outside this domain, EMD badly separates the two tones.

Fig. 7: Limits of agreement (in %) of Bland & Altman statistical tests between estimated SV<sub>CEEMD</sub> and reference SV<sub>ref</sub> for the 143 simulations. f is the frequency ratio (<f>f/h</f>) and α is the amplitude ratio (<a>α/hα</a>).

IV. CONCLUSION

This study has assessed two EMD algorithms applied to simulated CR signals. From each of these decompositions, the heart volume has been reconstructed based on periodicity considerations and the corresponding SV have been derived from. Afterwards the adequacy of the resulting SV with the reference has been evaluated statistically onto the whole range of physiological respiratory and cardiac parameters of the model. CEEMD turns out to be better than EMD, since CEEMD is able to extract the cardiac component from a signal mixing cardiac and respiratory information, in a larger domain of amplitude and frequency ratios than EMD. Further investigations may be based on simulated signals displaying more complex behaviors including non-stationarities and one-off events.

This work provides an innovative tool dedicated to CR signals including signal simulation, comparison of performance between time-scale decomposition methods and their validation. This last feature ensures the extraction of physiologically meaningful modes.

REFERENCES